1.4 - Functions

~~↳ consider a specific parking~~ meter. For a specific input (quarter, nickel, dime, etc), you get a specific output (20 min, 2 min, 8 min, etc.). It is useful to know that for a specific input, there is exactly one specific output. If, for example, you put in a quarter & sometimes you got 20 minutes & other times you get 3 minutes, the parking meter wouldnt be very useful.

↳ This is the idea of a function. A function takes inputs and assigns to each input exactly one output. That means if you know the input, you know exactly what output the function will spit out. There is no ambiguity.

→ Consider the equation $y = x^2$. For a specific x-value input, we know exactly what the y-value output will be.

→ we say that y is a function of x and write

$$y = f(x) = x^2$$

↳ f is the name of the function

↳ we say "f of x". This is not multiplication

↳ Functions work like this:

input → rule $f(x)$ → output

independent variable → x y ← dependent variable

3 ingredients:

i) Domain — set of all possible inputs

ii) Range — set of all possible outputs

iii) Rule — tells us what to do with input to compute output.

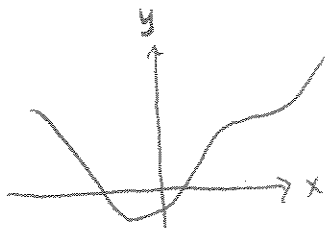
Ex: Is it a function?

a)

input	output
1	12
2	11
3	8
3	2
6	1

→ No! There are 2 output corresponding to an input of 3.

b)



x is input, y is output

↳ Yes, the graph represents y as a function of x because for each input, x, there is exactly 1 output, y.

c) Consider again $y = x^2$. Is x a function of y?

↳ solve for x: $\pm\sqrt{y} = x$ or $x = \pm\sqrt{y}$

↳ No! x is not a function of y because for each input, y, there are 2 outputs. For example, if $y=4$, the outputs, x, are both 2 and -2.

d) consider the equation $2x+3y=1$. Is y a function of x?

↳ solve for y: $3y = -2x+1 \Rightarrow y = -\frac{2}{3}x + \frac{1}{3}$

↳ Yes! for each x input, there is exactly 1 y output

→ x: independent variable, y: dependent variable

→ Is x a function of y?

↳ solve for x: $2x = -3y+1 \Rightarrow x = -\frac{3}{2}y + \frac{1}{2}$

↳ yes! For each y input, there is exactly one x output.

→ y: independent variable, x: dependent variable.

→ Consider the function $f(x) = 2x^3 - 1$

↳ input: x output: $f(x)$ → usually plot as y.

→ If we say $f(2)$ - "f of 2" - we mean the value of the function when $x=2$.

↳ Just plug in 2 for x.

Ex a) $f(2) = 2(2)^3 - 1$
$= 16 - 1$
$= 15$

b) $f(-3) = 2(-3)^3 - 1$
$= 2(-27) - 1$
$= -54 - 1$
$= -55$

c) $f(s) = 2s^3 - 1$

d) $f(x-1) = 2(x-1)^3 - 1$
$= 2(x-1)(x-1)(x-1) - 1$
$= 2(x^2 - 2x - 1)(x-1) - 1$
$= 2(x^3 - x^2 - 2x^2 + 2x - x + 1) - 1$
$= 2x^3 - 6x^2 + 2x + 1$

Ex: Let $g(x) = \begin{cases} \frac{x}{2} + 1, & x \leq 1 \\ 3x + 2, & x > 1 \end{cases}$ → This is called a piecewise-defined function

a) $f(3) = 3(3) + 2 = 11$ $f(-1) = -\frac{1}{2} + 1 = \frac{1}{2}$

b) $f(1) = \frac{1}{2} + 1 = \frac{3}{2}$ $f(\frac{4}{3}) = 3(\frac{4}{3}) + 2 = 6$

↳ we said that the domain of a function was the set of all possible inputs.

Ex Find the domain:

a) $f(x) = \frac{1}{x(x^2-4)}$; → can't divide by zero, so domain is all $\mathbb{R}$ except $x=0, x=2, x=-2$

$f(x) = \frac{1}{x(x-2)(x+2)}$

b) $g(x) = \sqrt{1-3x}$ → can't take square root of a negitive number, so need

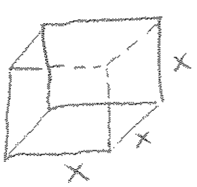
$1-3x \geq 0 \Rightarrow 1 \geq 3x \Rightarrow \boxed{\frac{1}{3} \geq x}$

$\boxed{(-\infty, 1/3]}$

↳ we can explicitly limit the domain of a function.

Ex Express volume of cube as a function of the length of its side.

$V(x) = x^3 \;, \; x > 0$



↳ we explicitly limit the domain to only include positive numbers.

Ex Express the area of a circle as a function of its circumference.

$A(r) = \pi r^2$ → function of r

$C = 2\pi r$ → Solve for r : $r = \frac{C}{2\pi}$

→ Plug into $A(r)$: $A(C) = \pi\left(\frac{C}{2\pi}\right)^2 \Rightarrow A(C) = \frac{\pi C^2}{4\pi^2} \Rightarrow \boxed{A(C) = \frac{C^2}{4\pi}}$

function of circumference