

1.3 - Linear Equations in 2 Variables

①

↳ Linear equations are the easiest ones to graph. Their graphs are just straight lines.

↳ The slope intercept form of a straight line is $y = mx + b$

↳ when $x=0$, $y=b$, so b is the y -intercept

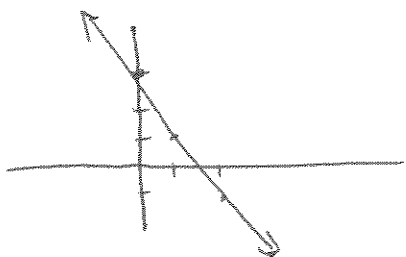
↳ m is the slope. The slope tells how steep a line is, and is given by

$$m = \frac{\text{rise}}{\text{run}} \quad \text{or} \quad m = \frac{\text{change in } y}{\text{change in } x}$$

If $m = \frac{2}{1}$, the line goes up 2 units for every 1 unit to the right

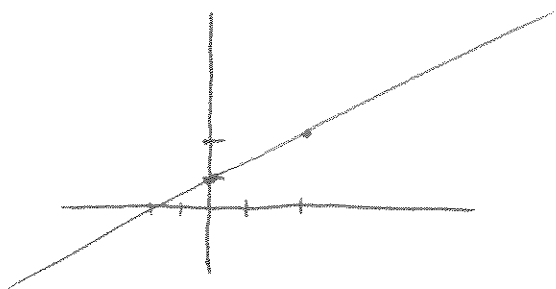
If $m = -\frac{1}{3}$, the line goes down 1 unit for every 3 units to the right.

Ex sketch $y = -2x + 3$



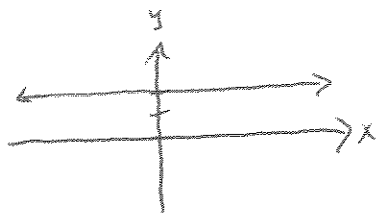
Ex Sketch $y = \frac{1}{2}x + 1$

(2)



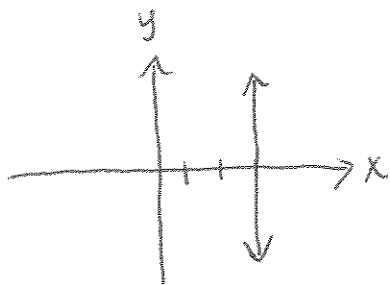
↳ A horizontal line has a slope of zero.

↳ $y = 2$ is a horizontal line. The y-value is 2 no matter what the x-value is.



↳ A vertical line has an undefined (or infinite) slope.

↳ $x = 3$ is a vertical line. The x value is 3 no matter what the y value is.



Finding a Slope

Ex Find the slope of the line connecting $(-2, 3)$ and $(2, -1)$

$$\rightarrow \text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x}$$

③

\hookrightarrow y goes from 3 to -1, a change of $-1-3 = -4$

\hookrightarrow x goes from -2 to 2, a change of $2-(-2) = 4$

\hookrightarrow The slope is $m = \frac{-4}{4} = -1$

\rightarrow For any 2 points (x_1, y_1) and (x_2, y_2) , the slope of the line connecting them is

$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{or} \quad m = \frac{y_1 - y_2}{x_1 - x_2}$$

\rightarrow They're both the same number. Just be consistent!

Ex Find slope of line connecting

a) $(-3, 4)$ and $(2, 1)$

$$\hookrightarrow m = \frac{1-4}{2-(-3)} = \frac{-3}{5}$$

b) $(-3, 2)$ and $(2, 2)$

$$\hookrightarrow m = \frac{2-2}{2-(-3)} = \frac{0}{5} = 0 \rightarrow \text{horizontal line}$$

c) $(2, 3)$ and $(2, -1)$

$$\hookrightarrow m = \frac{-1-3}{2-2} = \frac{-4}{0} \rightarrow \text{undefined} \hookrightarrow \text{vertical line}$$

Ex Find the equation in slope intercept form of the line through $(1,2)$ and $(2,3)$

(4)

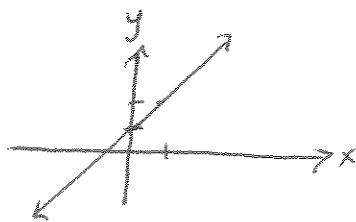
↳ first find slope: $m = \frac{3-2}{2-1} = \frac{1}{1} = 1$

so in $y = mx + b$, we have $y = x + b$

↳ both points satisfy the equation of the line. Use this to solve for b

$$2 = 1 + b \Rightarrow b = 1 \quad \text{or} \quad 3 = 2 + b \Rightarrow b = 1$$

⇒ Eqn is $y = x + 1$



↳ we could also use the point-slope form:

$$y - y_1 = m(x - x_1)$$

↳ x and y are variables, (x_1, y_1) is a point on the line, and m is the slope we found.

Parallel / Perpendicular Lines

↳ lines that are parallel have the same slope.

Ex: Find eqn. of line parallel to $2x - 3y = 1$, through the point $(-1, 2)$

↳ first find the slope: put $2x - 3y = 1$ in slope-intercept form

$$-3y = -2x + 1$$

$$\Rightarrow y = \frac{2}{3}x - \frac{1}{3} \quad \rightarrow \text{slope is } \frac{2}{3}$$

→ so our line is $y = \frac{2}{3}x + b$

↳ use the point to find b.

$$2 = \frac{2}{3}(-1) + b$$

$$\Rightarrow b = 2 + \frac{2}{3}$$

$$\Rightarrow b = \frac{6}{3} + \frac{2}{3}$$

$$\Rightarrow b = \frac{8}{3}$$

→ so $y = \frac{2}{3}x + \frac{8}{3}$

↳ Perpendicular lines have slopes that are negative reciprocals of each other

Ex Find eqn. of line perpendicular to $4x - y - 2 = 0$ through the point $(1, 1)$

↳ find slope $4x - y - 2 = 0 \Rightarrow -y = -4x + 2 \Rightarrow y = 4x - 2$
↳ slope = 4

↳ slope of our line is $m = -\frac{1}{4}$

$y = -\frac{1}{4}x + b$. use point to find b

$$1 = -\frac{1}{4}(1) + b \Rightarrow 1 + \frac{1}{4} = b \Rightarrow \frac{4}{4} + \frac{1}{4} = b \Rightarrow \frac{5}{4} = b$$

Thus $y = -\frac{1}{4}x + \frac{5}{4}$

Ex A company pays \$11.50 per hour plus \$0.75 per unit produced. Write linear equation for hourly wage, w , in terms of # of units, x , produced per hour. (6)

↳ if you make x unit per hour, you get $0.75x$ in addition to the flat hourly rate of \$11.50. Thus the full wage is

$$w = 0.75x + 11.5$$

Ex A discount store is giving a 15% discount on all items. Write equation for sale price, S , in terms of original list price, L .

→ Sale price = original price - discount
→ Original price = L
→ discount = $0.15L$

so $S = L - 0.15L \Rightarrow S = 0.85L$