1.2 - Graphs of Equations

$\hookrightarrow$ A graph is a way to visualize an equation in 2 variables.

$\hookrightarrow$ Each point on a graph corresponds to a pair of values that solves the equation.

$\hookrightarrow$ we draw graphs by choosing values for one off the variables and calculating the corresponding value for the other variable that solves the equation.
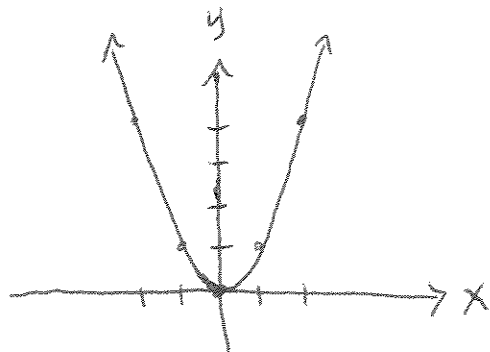
Ex Plot $y = 2x + 1$

$\hookrightarrow$ we choose x-values, calculate the corresponding y-values, plot the points, & connect the dots.

X	$y = 2x + 1$
-2	-3
-1	-1
0	1
1	3
2	5
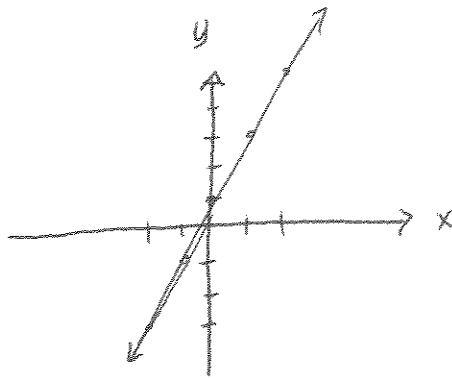
Plot: $y = x^2$

X	$y = x^2$
-2	4
-1	1
0	0
1	1
2	4

$\to$ This shape is called a parabola

$\to$ This graph is symmetric across the y-axis

$\hookrightarrow$ same y value for positive & negative x values

→ To get a sketch of an equation, it's often useful to figure out the points, called intercepts, where the graph crosses one of the axes.

→ x-intercept → Graph touches x-axis. y value is zero

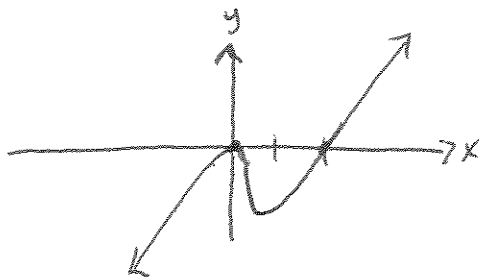y-intercept → Graph touches y-axis. x value is zero

Ex Find the intercepts of $y = 2x^3 - 4x^2$

x-intercepts → set $y=0$ and solve

$$0 = 2x^3 - 4x^2 \Rightarrow 0 = x^3 - 2x^2$$
$$\Rightarrow 0 = x^2(x-2)$$
$$\Rightarrow x=0, \; x=2$$

y-intercepts → set $x=0$ and solve

$$y = 2(0)^3 - 4(0)^2 \Rightarrow y=0$$

→ graph looks something like this.

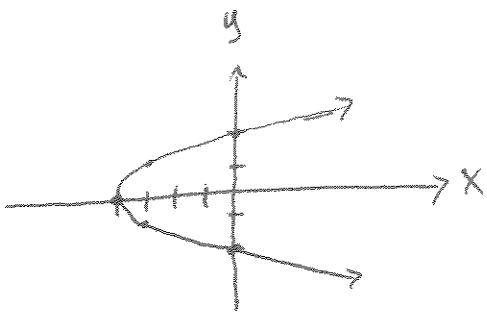Ex sketch a graph of $x = y^2 - 4$. Find the intercepts.

x-intercept : → set $y=0$. $\qquad x = 0^2 - 4 \Rightarrow x = -4$

y-intercepts: set $x=0$. $\qquad 0 = y^2 - 4$

$$\Rightarrow 0 = (y-2)(y+2)$$
$$\Rightarrow y = 2, -2$$

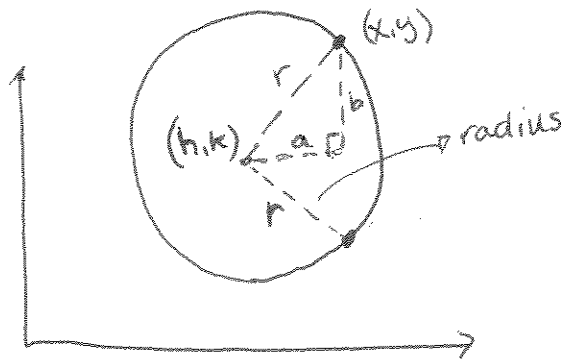| y	$x = y^2 - 4$
-2	0
-1	-3
0	-4
1	-3
2	0

↳ parabola on its side

↳ symmetric across x-axis

↳ same x value for positive & negative y-values.

Circles

↳ if you choose a point, (h,k) and draw all the points that are an equal distance from (h,k), you get a circle whose center is (h,k). the distance from the center to the edge is called the radius.

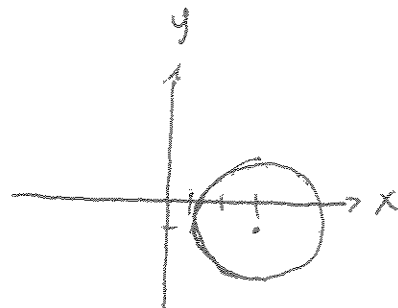distance a : x-h

distance b : y-k

so $r^2 = (x-h)^2 + (y-k)^2$

This is the standard form for a circle with radius r and center (h,k).

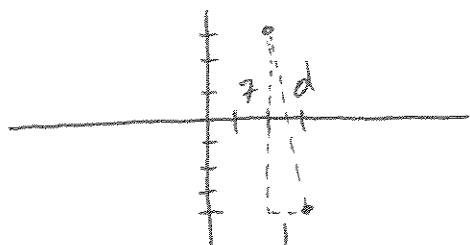EX Find the equation of a circle with center at (3,-1) and radius 2.

→ $h=3$, $k=-1$, $r=2$

so $(x-3)^2 + (y+1)^2 = 4$

EX You stand on the point $(2,3)$ and spin around while holding one end of a jumprope. The other end of the jumprope just nicks a post on the point $(3,-4)$. How long is the jumprope? The end of the jumprope traces a circle in the air. What is the equation of that circle?

$d^2 = 7^2 + 1^2$

$\Rightarrow d^2 = 49+1$

$\Rightarrow d^2 = 50$

$\Rightarrow d = \sqrt{50}$

$\Rightarrow d = \sqrt{25 \cdot 2}$

$\Rightarrow \boxed{d = 5\sqrt{2}}$

radius of circle is $\sqrt{50}$.

so equation is

$(x-2)^2 + (y-3)^2 = \left(\sqrt{50}\right)^2 \Rightarrow (x-2)^2 + (y-3)^2 = 50$