9.5 - Solving Exponential & Logarithmic Equations

→ To solve equations with exponentials and logarithms, we use inverse properties, properties of exponentials & properties of logarithms

Inverse Property

$\log_a a^x = x$, $a^{\log_a x} = x$

Properties of Exponentials

1) $a^m a^n = a^{m+n}$

2) $\frac{a^m}{a^n} = a^{m-n}$

3) $(a^m)^n = a^{mn}$

Properties of Logarithms

1) $\log_a u + \log_a v = \log_a uv$

2) $\log_a u - \log_a v = \log_a\left(\frac{u}{v}\right)$

3) $\log_a u^n = n \log_a u$

Ex Solve

$3^{x+2} = \frac{1}{27}$ → want to get x out of the exponent. use inverse property

$\log_3 3^{x+2} = \log_3 \frac{1}{27}$

$\Rightarrow x+2 = \log_3 \frac{1}{27}$

$\Rightarrow x+2 = \log_3 27^{-1}$

$\Rightarrow x+2 = -\log_3 27$

$\Rightarrow x+2 = -\log_3 3^3$

$\Rightarrow x+2 = -3\log_3 3$

$\Rightarrow x+2 = -3$

$\Rightarrow x = -5$

Check: $3^{-5+2} = 3^{-3}$

$= \frac{1}{3^3}$

$= \frac{1}{27}$ ✓

→ sometimes we don't get an answer as nice as the last one

check: $4^{3+\log_4 9 - 3} = 4^{\log_4 9}$

$= 9$ ✓

Ex Solve

$$4^{x-3} = 9$$

$\Rightarrow \log_4 4^{x-3} = \log_4 9$

$\Rightarrow x - 3 = \log_4 9$

$\Rightarrow x = 3 + \log_4 9$ → can't evaluate without a calculator

Ex Solve

$5 + e^{x+1} = 20$ → first get exponential by itself

$\Rightarrow e^{x+1} = 15$

$\Rightarrow \ln e^{x+1} = \ln 15$

$\Rightarrow x + 1 = \ln 15$

$\Rightarrow x = -1 + \ln 15$

$5 + e^{-1+\ln 15+1} = 5 + e^{\ln 15}$

$= 5 + 15 = 20$ ✓

→ we can solve equations with logarithms too.

Ex Solve

$$\log_3 x = 2$$

→ want to get the x out of the logarithm.
↳ use inverse property

$\Rightarrow 3^{\log_3 x} = 3^2$

$\Rightarrow x = 9$

Ex Solve

$2 + 3\log_3(x-1) = 8$ → get the log by itself first

$\Rightarrow 3\log_3(x-1) = 6$

$\Rightarrow \log_3(x-1) = 2$

$\Rightarrow 3^{\log_3(x-1)} = 3^2$

$\Rightarrow x-1 = 9$

$\Rightarrow x = 10$

check: $2 + 3\log_3(10-1)$

$= 2 + 3\log_3(9)$

$= 2 + 3\log_3(3^2)$

$= 2 + 6\log_3 3$

$= 2 + 6 = 8$ ✓

Ex Solve

$24 + e^{4-x} = 22$

$\Rightarrow e^{4-x} = -2$

$\Rightarrow \ln e^{4-x} = \ln(-2)$ → Can't take the log of a negative number!

no solution

Ex Solve

$\frac{5000}{(1.05)^x} = 250$ → get exponential by itself

$\Rightarrow 5000 = 250 \cdot (1.05)^x$

$\Rightarrow \frac{5000}{250} = (1.05)^x$

$\Rightarrow 20 = (1.05)^x$

$\Rightarrow \log_{1.05} 20 = \log_{1.05}(1.05)^x$

$\Rightarrow \log_{1.05} 20 = x$

Ex Solve

$\frac{2}{3}\ln(x+1) = -1$

$\Rightarrow \ln(x+1) = -\frac{3}{2}$

$\Rightarrow e^{\ln(x+1)} = e^{-3/2}$

$\Rightarrow x+1 = e^{-3/2}$

$\Rightarrow x = -1 + e^{-3/2}$

Note: This is not correct:

$5^{\log_5 x} - 5^{\log_5 4} = 5^2$

$\Rightarrow x - 4 = 25$

$\Rightarrow x = 29$

This is NOT the correct answer

Ex Solve

$\log_5 x - \log_5 4 = 2$ → combine logs first

$\Rightarrow \log_5\left(\frac{x}{4}\right) = 2$

$\Rightarrow 5^{\log_5\left(\frac{x}{4}\right)} = 5^2$

$\Rightarrow \frac{x}{4} = 25$

$\Rightarrow x = 100$

check: $\log_5 100 - \log_5 4 = \log_5 \frac{100}{4}$

$= \log_5 25$

$= \log_5 5^2$

$= 2\log_5 5$

$= 2$ ✓

Ex Solve

$\log_3 2x + \log_3(x-1) - \log_3 4 = 1$

$\Rightarrow \log_3(2x(x-1)) - \log_3 4 = 1$

$\Rightarrow \log_3 \frac{2x(x-1)}{4} = 1$

$\Rightarrow 3^{\log_3 \frac{2x(x-1)}{4}} = 3^1$

$\rightarrow \frac{2x(x-1)}{4} = 3$

$\Rightarrow 2x(x-1) = 12$

$\rightarrow 2x^2 - 2x - 12 = 0$

$\Rightarrow x^2 - x - 6 = 0$

$\Rightarrow (x-3)(x+2) = 0$

$\Rightarrow x = 3, -2$

check: $\log_3 6 + \log_3 2 - \log_3 4$

$= \log_3 \frac{6\cdot 2}{4} = \log_3 3 = 1$ ✓

$\log_3(-4) + \log_3(-3) - \log_3 4 = ?$ NO!

→ can't take the log of a negative number!

$\boxed{x=3}$ is the only solution

Supplementary Problems

7, 9, 11, 23, 29, 41, 43, 51, 57, 61, 67, 77, 79, 81, 83, 85, 87, 95, 99, 105, 107, 113, 117