9.4 - Properties of Logarithms

→ Recall the properties of exponents:

$$a^m a^n = a^{m+n}, \quad \frac{a^m}{a^n} = a^{m-n}, \quad (a^m)^n = a^{mn}$$

→ The properties of logarithms come from these, but they look a bit different.

→ First remember that $f(x) = \log_a x$ and $g(x) = a^x$ are inverses. So then

$$\log_a(a^x) = x \quad \text{and} \quad a^{(\log_a x)} = x$$

→ Consider the following

1) $$a^{\log_a u + \log_a v} = a^{\log_a u} \cdot a^{\log_a v} = uv = a^{\log_a uv}$$

so then $$\boxed{\log_a uv = \log_a u + \log_a v}$$

2) $$a^{\log_a u - \log_a v} = \frac{a^{\log_a u}}{a^{\log_a v}} = \frac{u}{v} = a^{\log_a \frac{u}{v}}$$

so then $$\boxed{\log_a \frac{u}{v} = \log_a u - \log_a v}$$

} base must be the same

3) $$a^{n\log_a u} = \left(a^{\log_a u}\right)^n = u^n = a^{\log_a u^n}$$

so then $$\boxed{\log_a u^n = n \cdot \log_a u}$$

Note: $\log_a(u+v) \neq \log_a u + \log_a v$

$n \cdot \log_a u \neq \log_a(nu)$

Ex Evaluate

a) $\log_3 81 = \log_3 3^4 = 4\log_3 3 = 4$

b) $\log_7\left(\frac{1}{49}\right)^3 = 3\log_7 \frac{1}{49} = 3\log_7 49^{-1} = -3\log_7 49 = -3\log_7 7^2$
$= -6\log_7 7$
$= -6$

c) $\log_6 2 + \log_6 3 = \log_6(2\cdot 3) = \log_6 6 = 1$

d) $\log_5 50 - \log_5 2 = \log_5\left(\frac{50}{2}\right) = \log_5(25) = \log_5 5^2$
$= 2\log_5 5 = 2$

e) $\ln e^5 - \ln e^2 = \ln\left(e^5/e^2\right) = \ln e^{5-2} = \ln e^3 = 3\ln e = 3$
or $\ln e^5 - \ln e^2 = 5\ln e - 2\ln e = 5 - 2 = 3$

Ex Expand the expression

a) $\log_2 3x = \log_2 3 + \log_2 x$

b) $\log_3 \sqrt[3]{5y} = \log_3 (5y)^{1/3} = \frac{1}{3}\log_3 5y = \frac{1}{3}\left(\log_3 5 + \log_3 y\right)$

c) $\ln\left(\frac{\sqrt{x}}{x+9}\right) = \ln\sqrt{x} - \ln(x+9) = \ln x^{1/2} - \ln(x+9) = \frac{1}{2}\ln x - \ln(x+9)$

can't expand this to $\ln x + \ln 9$

d) $\log_8\left[(x-y)^3 z^6\right] = \log_8 (x-y)^3 + \log_8 z^6$
$= 3\log_8(x-y) + 6\log_8 z$

Ex Condense into a single logarithm

a) $\log_6 12 - \log_6 y = \log_6\left(\frac{12}{y}\right)$

b) $12\ln z = \ln z^{12}$

c) $\log_{10} 6 - 3\log_{10} z = \log_{10} 6 - \log_{10} z^3$
$= \log_{10}\left(\frac{6}{z^3}\right)$

Deal with #'s multiplying the log first!

d) $\frac{1}{4}\log_6(x+1) - 5\log_6(x-4) = \log_6(x+1)^{\frac{1}{4}} - \log_6(x-4)^5$

$= \log_6 \frac{(x+1)^{\frac{1}{4}}}{(x-4)^5}$

e) $2\log_5(x+y) + 3\log_5 w$

$= \log_5(x+y)^2 + \log_5 w^3 = \log_5\left((x+y)^2 w^3\right)$

True or False?

a) $\log_2 8x = 3 + \log_2 x$

True! $\log_2 8x = \log_2 8 + \log_2 x = \log_2 2^3 + \log_2 x$

$= 3\log_2 2 + \log_2 x$

$= 3 + \log_2 x$

b) $\log_3(u+v) = \log_3 u + \log_3 v$

False! $\log_3(uv) = \log_3 u + \log_3 v$

c) $\log_3(u+v) = (\log_3 u)(\log_3 v)$

False! → neither side can be simplified

d) $\frac{\log_6 10}{\log_6 3} = \log_6\left(\frac{10}{3}\right)$

False!

e) $\frac{\log_6 10}{\log_6 3} = \log_6 10 - \log_6 3$

False!

Supplementary Problems

1, 3, 5, 7, 13, 15, 19, 21, 47, 49, 59, 61, 63, 69, 71, 75, 77, 89, 91, 95, 97, 101

103, 105, 107