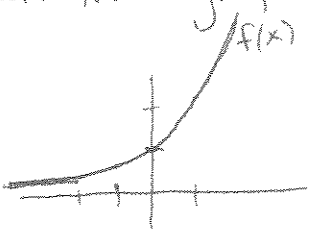
9.3 - Logarithmic Functions

→ Recall the graph of an exponential function, say $f(x)=2^x$



→ this function passes the horizontal line test. Thus it is 1-to-1 and it has an inverse.

→ The inverse of an exponential function is a logarithmic function.

$$y = \log_a x \quad \text{if} \quad x = a^y$$

→ The function $f(x) = \log_a x$ is the logarithmic function with base a.

→ To find $\log_a x$, we ask "what power must I raise a to in order to get x"

Ex a) $\log_2 8 = 3$ because $2^3 = 8$

b) $\log_5 1 = 0$ because $5^0 = 1$

c) $\log_{10} \frac{1}{10} = -1$ because $10^{-1} = \frac{1}{10}$

d) $\log_3 (-1)$ → no real number because 3 to a power will be positive, not negative

e) $\log_4 0$ → ~~not a real number~~ undefined Can't raise 4 to a power & get zero

Logarithm Facts

1) $\log_a 1 = 0$ because $a^0 = 1$

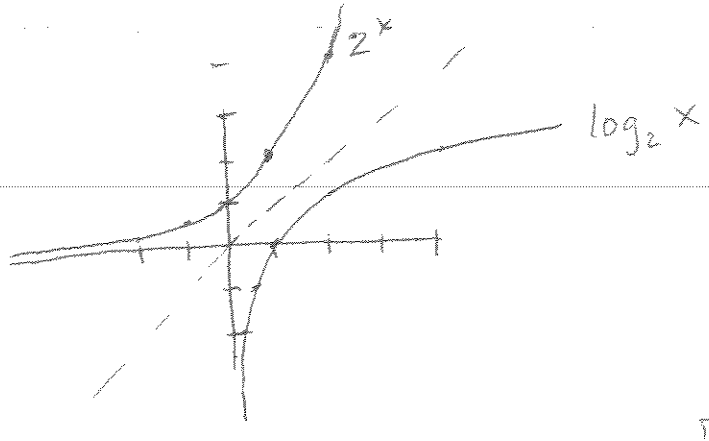
2) $\log_a a = 1$ because $a^1 = a$

3) $\log_a a^x = x$ they are inverses

→ So what does the graph of a logarithm look like?

↳ It's the inverse of an exponential function, so we'll use that fact

→ Consider $f(x) = \log_2 x$ → It's a reflection across $y = x$ of $g(x) = 2^x$



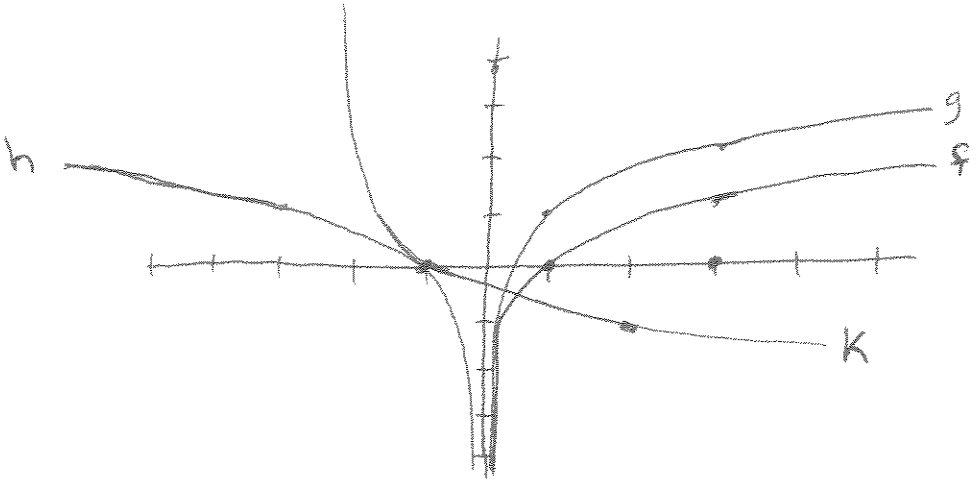
for $a > 1$, $\log_a x$ always has this general shape.

Domain: $(0, \infty)$
Range: $(-\infty, \infty)$
Intercept: $(1, 0)$
increasing

→ Like all other functions, we can look at translations/reflections of logarithms

Ex Plot $f(x) = \log_3 x$, $g(x) = \log_3 x + 1$, $h(x) = \log_3(-x)$

$k(x) = -\log_3(x+2)$



→ Remember that weird number $e \approx 2.71828\ldots$?

↳ if we use it as a base for a logarithm; $\log_e x$, we call it the natural logarithm and write $\ln x$

→ So $f(x) = \log_e x = \underbrace{\ln x}$

↳ this just means a logarithm with a base of e.

→ It works just like other logarithms. i.e.

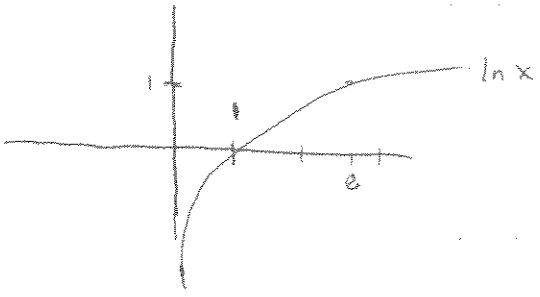
1) $\ln 1 = 0$ since $e^0 = 1$
2) $\ln e = 1$ since $e^1 = e$
3) $\ln e^x = x$ since $e^x = e^x$ ($\ln x$ and e^x are inverses)

Ex Evaluate

a) $\ln e^3 = 3$

b) $\ln \frac{1}{e} = -1$ since $\ln \frac{1}{e} = \ln e^{-1}$

plotting $\ln(x)$:



Change of Base formula

→ This is just a practical formula to know because many calculators can only evaluate $\ln x$ or $\log_{10} x$.

In general: $\log_a x = \frac{\log_b x}{\log_b a}$. if $b = 10$, $\log_a x = \frac{\log_{10} x}{\log_{10} a}$

if $b = e$, $\log_a x = \frac{\ln x}{\ln a}$

Ex To evaluate $\log_3 5$ on many calculators, you can do

$$\frac{\log_{10} 5}{\log_{10} 3} \quad \text{or} \quad \frac{\ln 5}{\ln 3}$$

Final Note: Many times when you see $\log x$, it means $\log_{10} x$. Sometimes $\log x$ means $\ln x$. It depends on the program, so you have to check. In WebWork, use $\ln(\,)$ for natural logarithm and $\text{logten}(\,)$ for $\log_{10}(\,)$.

Supplementary Problems

1, 3, 5, 7, 9, 13, 15, 17, 19, 25, 27, 29, 31, 35, 37, 57, 59, 61, 63, 65, 77, 79, 81