9.1 - Exponential Functions

→ We've worked a lot with things like $f(x) = x^3$, $f(x) = x^{1/4}$, etc.

↳ The variable is raised to a constant power.

→ We can also have things like $f(x) = 2^x$ → base

→ The variable is in the exponent

→ This is an exponential function with a base of 2

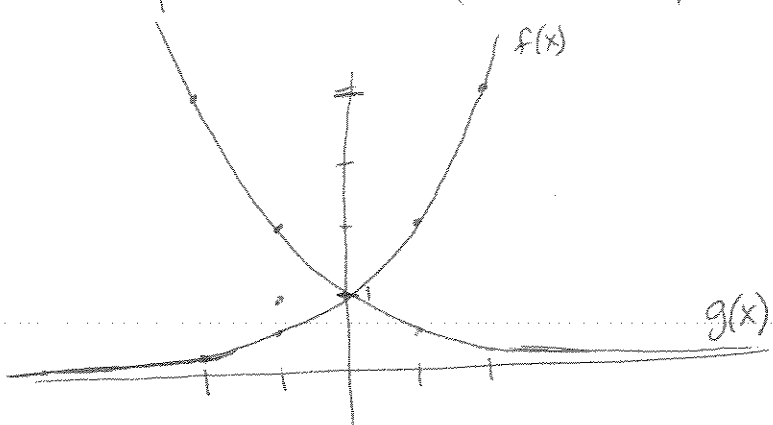
→ an exponential function has a positive base

what does ~~an~~ exponential function look like?

↳ It depends whether the base is greater than 1 or less than 1.

↳ We can plot points to get an idea of the shape.

	-2	-1	0	1	2
$f(x) = 2^x$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4
$g(x) = \left(\frac{1}{2}\right)^x$	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$



if the base, a, of $f(x) = a^x$ is >0, f(x) goes up from left to right (increasing)

if $a < 1$, $f(x) = a^x$ goes down from left to right. (decreasing)

→ Plotting these functions often shows up on the final!

→ notice tha f(x) and g(x) above both pass through (0, 1) since $a^0 = 1$ for all $a \neq 0$.

→ How do we do operations with exponential functions?

↳ The rules are the same as the rules for exponents.

Rules

1) $a^x \cdot a^y = a^{x+y}$ → add exponents when multiplying w/ same base

2) $\dfrac{a^x}{a^y} = a^{x-y}$

3) $(a^x)^y = a^{xy}$

4) $a^{-x} = \dfrac{1}{a^x} = \left(\dfrac{1}{a}\right)^x$

→ You can evaluate exponential functions at some values, but you often need a calculator.

Ex let $f(x) = 3^x$

a) $f(1) = 3^1 = 3$

b) $f(-2) = 3^{-2} = \dfrac{1}{3^2} = \dfrac{1}{9}$

c) $f(\frac{2}{3}) = 3^{2/3} = \left(\sqrt[3]{3}\right)^2$ → need a calculator!

> what is the domain of an exponential function?

→ from the graph, we see that the domain is all real numbers, $\mathbb{R}$.

> what is the range?

→ positive numbers ; $(0, \infty)$

> let's look back at $g(x) = \left(\frac{1}{2}\right)^x$

→ using the rules, $g(x) = \left(\frac{1}{2}\right)^x$

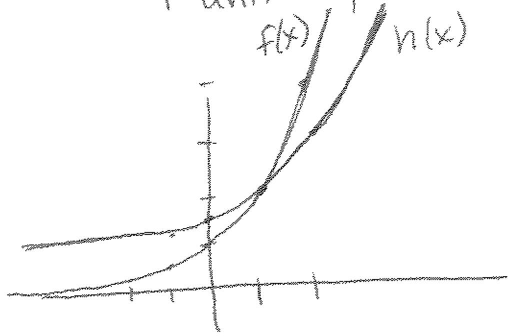
$= \dfrac{1}{2^x} = 2^{-x}$

→ any time $a<1$, we can do this and end up with an $a>1$ and an exponent with a negitive sign.

→ If we do this we see that $f(x) = a^x$ with $a > 1$ is always increasing and $f(x) = a^{-x}$ with $a > 1$ is always decreasing.

→ what about something like $h(x) = 2^{x-1} + 1$

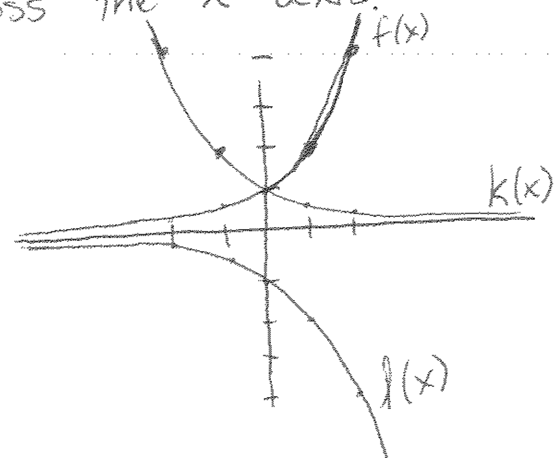
↳ This looks like $f(x) = 2^x$ but shifted 1 unit right and 1 unit up.



→ the shifting rules are the same as what we've seen previously.

→ what about $k(x) = 2^{-x}$ and $l(x) = -2^x$

→ $k(x)$ is $f(x)$ flipped across the y-axis. $l(x)$ is $f(x)$ flipped across the x axis.



→ It gets quite confusing when you combine these things together. If you're having trouble, remember you can always choose x-values, calculate the corresponding y-values, and connect the dots.

~~Supplementary problems~~

- Often, we talk about exponential functions with a very specific base, a number called $e \approx 2.71828\ldots$

- e is an irration number like π that seems almost magical because it just shows up in a bunch of different mathematical applications. your calculator probably has an e button.
- Just remember that $f(x)=e^x$ is just a normal exponential function with a specific base, the number e.
- $f(x)=e^x$ is often called the natural exponential.
- we'll see $f(x)=e^x$ show up again in a few days when we talk about compound interest.
- $f(x)=e^x$ has some really nice properties that show up in calculus.

Supplementary Problems pp.. 586-587

1, 3, 5, 7, 9, 15, 17, 21, 31, 33, 37, 39, 43, 63, 65, 67