8.5 - Applications of Quadratic Equations

→ Consider a falling object

h: height in feet
V_0: initial vertical velocity
h_0: height at time zero

height at time t is given by

$$h(t) = -16t^2 + V_0 t + h_0$$

Ex You throw a rock straight up. You release it from your hand at an initial height of 6 feet and initial velocity of 16 feet/second

→ how long does it stay in the air?
→ how high does it go?

$$h(t) = -16t^2 + 16t + 6$$

$h=0$ when rock hits the ground. solve for t when $h=0$

$$0 = -16t^2 + 16t + 6$$

$$0 = \quad t^2 - t - \_\_$$

$$\Rightarrow t^2 - t =$$

$$\Rightarrow t^2 - 1 + \left(-\tfrac{1}{2}\right)^2 = \tfrac{3}{8} + \left(-\tfrac{1}{2}\right)^2$$

$$\Rightarrow \left(t - \tfrac{1}{2}\right)^2 = \tfrac{3}{8} + \tfrac{1}{4}$$

$$\Rightarrow \left(t - \tfrac{1}{2}\right)^2 = \tfrac{5}{8}$$

$$\Rightarrow t - \tfrac{1}{2} = \pm\sqrt{\tfrac{5}{8}} \cdot \tfrac{\sqrt{8}}{\sqrt{8}}$$

$$\Rightarrow t - \tfrac{1}{2} = \pm\tfrac{\sqrt{40}}{8}$$

$$\Rightarrow t - \tfrac{1}{2} = \pm\tfrac{2\sqrt{10}}{8}$$

$$\Rightarrow t - \tfrac{1}{2} = \pm\tfrac{\sqrt{10}}{4}$$

$$\Rightarrow t = \tfrac{1}{2} \pm \tfrac{\sqrt{10}}{4}$$

→ only take the positive solution

→ How high does it go?

$h(t) = -16t^2 + 16t + 6$ is a parabola. If we put it in standard form, we'll be able to see where the vertex is + the max height of the ~~vertex~~ rock.

$$h(t) = -16\left(t^2 - t - \frac{6}{16}\right)$$

$$= -16\left(t^2 - t - \frac{3}{8}\right)$$

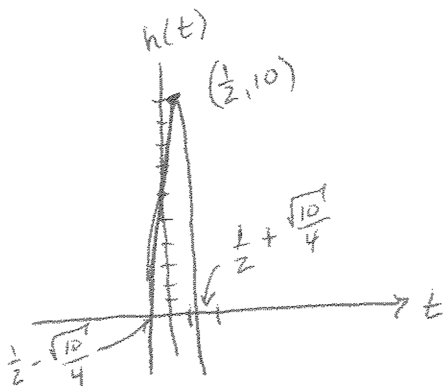
$$= -16\left(t^2 - t + \left(-\frac{1}{2}\right)^2 - \left(-\frac{1}{2}\right)^2 - \frac{3}{8}\right)$$

$$= -16\left(\underbrace{t^2 - t + \left(-\frac{1}{2}\right)^2}_{\text{perfect square}} - \frac{1}{4} - \frac{3}{8}\right)$$

$$= -16\left(\left(t - \frac{1}{2}\right)^2 - \frac{5}{8}\right)$$

$$= -16\left(t - \frac{1}{2}\right)^2 + 10$$

vertex is $(1/2, 10)$, so rock has maximum height of 10 feet at $t = 1/2$ seconds

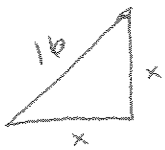


→ Note: this problem is slightly unrealistic in that it ignores air resistance

Ex Suppose you throw the rock at a 45° angle so the horizontal and vertical components of the initial vecocity are identical. When will it hit the ground? How far will it fly?

Initial height still 6 feet

initial velocity still 16 feet/second



$$16^2 = x^2 + x^2$$

$$16^2 = 2x^2$$

$$\sqrt{16^2} = \pm\sqrt{2x^2}$$ → just take the +

$$16 = x\sqrt{2}$$

$$x = \frac{16}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$

$$x = \frac{16\sqrt{2}}{2} = 8\sqrt{2}$$

horizontal component of velocity is $8\sqrt{2}$ feet/second + so is vertical component

So now $h(t) = -16t^2 + 8\sqrt{2}\,t + 6$. hits ground when $h = 0$

$$0 = -16t^2 + 8\sqrt{2}\,t + 6$$

$$\Rightarrow 0 = t^2 - \frac{\sqrt{2}}{2}t - \frac{6}{16}$$

$$\Rightarrow t^2 - \frac{\sqrt{2}}{2}t = \frac{3}{8}$$

$$\Rightarrow t^2 - \frac{\sqrt{2}}{2}t + \left(-\frac{\sqrt{2}}{4}\right)^2 = \frac{3}{8} + \left(-\frac{\sqrt{2}}{4}\right)^2$$

$$\Rightarrow \left(t - \frac{\sqrt{2}}{4}\right)^2 = \frac{3}{8} + \frac{2}{16}$$

$$\Rightarrow \left(t - \frac{\sqrt{2}}{4}\right)^2 = \frac{3}{8} + \frac{1}{8} =$$

$$\Rightarrow \left(t - \frac{\sqrt{2}}{4}\right)^2 = \frac{1}{2}$$

$$\Rightarrow t - \frac{\sqrt{2}}{4} = \pm\sqrt{\frac{1}{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$

$$\Rightarrow t - \frac{\sqrt{2}}{4} = \pm\frac{\sqrt{2}}{2}$$

$$\Rightarrow t = \frac{\sqrt{2}}{4} \pm \frac{\sqrt{2}}{2}$$

$$\Rightarrow t = \frac{\sqrt{2} \pm 2\sqrt{2}}{4}$$ take positive time

$$\boxed{t = \frac{3\sqrt{2}}{4}}$$

→ how far does it fly?

distance = rate · time

horizontal velocity = $8\sqrt{2}$ feet/second

so then $d = \frac{3\sqrt{2}}{4} \cdot 8\sqrt{2}$

$= \frac{24(2)}{4} = 12$ feet

→ hmmm... maybe you should practice your throwing.