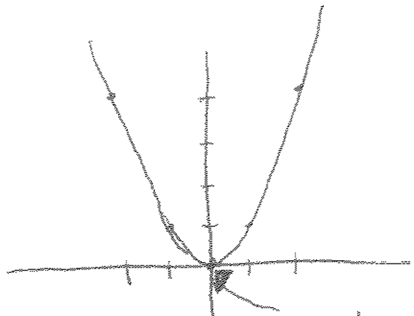
8.4 - Graphs of Quadratic Functions

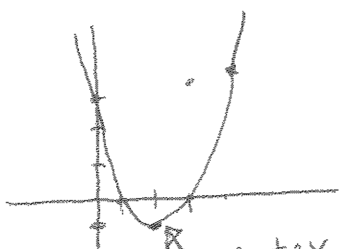
→ Recall that the graph of the function $f(x) = x^2$ is a parabola.
and looks like



vertex at (0,0)

→ choose input values, x, and calculate the corresponding y values. Then connect the points

→ Also recall that $f(x) = (x-2)^2 - 1$ is the same parabola shifted 2 units to the right and 1 unit down



vertex at (2,-1)

→ The standard form of a parabola is $f(x) = a(x-h)^2 + k$

→ a determines how wide or skinny the opening is

→ sign of a determines whether the parabola opens up or down

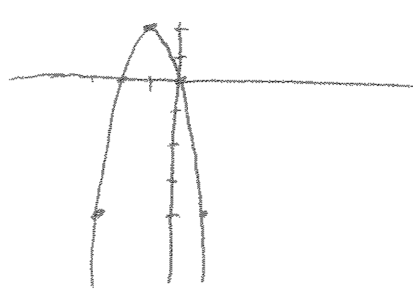
→ (h,k) are the coordinates of the vertex.

EX $f(x) = -2(x+1)^2 + 2$

→ opens down

→ vertex at (-1,2)

→ opening twice as narrow



→ we can get this graph by just plotting points even if we don't remember the rules.

Ex What about a general quadratic equation that isn't written in the standard form of a parabola?

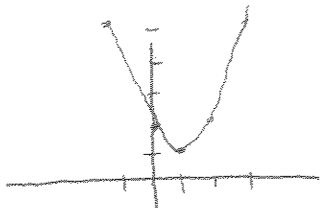
$f(x) = x^2 - 2x + 2$

Notice $x^2 - 2x + 1 = (x-1)^2$ is a perfect square. And we can write

$$f(x) = x^2 - 2x + 1 - 1 + 2$$

$$= x^2 - 2x + 1 + 1$$

$$= \underbrace{(x-1)^2 + 1}_{\text{standard form. Easy to graph}} \quad \rightarrow \text{parabola with vertex } (1,1)$$



Ex leading coefficient not 1

$f(x) = 2x^2 + 6x + 2$ → can't divide by 2 since it's not an equation
↳ Instead, factor out the 2

$f(x) = 2(x^2 + 3x + 1)$

→ now complete the square on $x^2 + 3x$

$$f(x) = 2\left(x^2 + 3x + \left(\tfrac{3}{2}\right)^2 - \left(\tfrac{3}{2}\right)^2 + 1\right)$$

$$f(x) = 2\left(\underbrace{x^2 + 3x + \left(\tfrac{3}{2}\right)^2}_{\text{perfect square}} - \tfrac{9}{4} + 1\right)$$

$$f(x) = 2\left(\left(x + \tfrac{3}{2}\right)^2 - \tfrac{5}{4}\right) \quad \rightarrow \text{now distribute}$$

$$f(x) = 2\left(x + \tfrac{3}{2}\right)^2 - \tfrac{5}{2}$$

Vertex at $\left(-\frac{3}{2}, -\frac{5}{2}\right)$

opens up. Twice as narrow