3.1 - Solving Quadratic Equations: Factoring and Special forms

› Recall that a quadratic equation is an equation of the form $ax^2+bx+c=0$ with $a\neq 0$

we've seen in previously that one way to solve equations of this form is by factoring

Ex Solve $x^2-7x=18$

$$\Rightarrow x^2-7x-18=0$$
$$\Rightarrow (x-9)(x+2)=0$$
$$\Rightarrow \boxed{x=9,-2}$$

› This only works in special cases, so we need more solution methods

Square Root Property:

Consider the quadratic equation $u^2=d$

We could solve this like a difference of squares:

$$u^2-d=0$$
$$\Rightarrow (u-\sqrt{d})(u+\sqrt{d})=0$$
$$\Rightarrow u=\pm\sqrt{d}$$

→ we see we could solve $u^2=d$ by taking the square root of both sides. But we have to remember that we get both a positive and negative solution

Ex Solve $2x^2=16$

$$\Rightarrow x^2=8$$
$$\Rightarrow x=\pm\sqrt{8}$$
$$\Rightarrow \boxed{x=\pm 2\sqrt{2}}$$

Ex Solve $(x+3)^2 - 5 = 0$

$\Rightarrow (x+3)^2 = 5$

$\Rightarrow x+3 = \pm\sqrt{5}$

$\Rightarrow \boxed{x = -3 \pm \sqrt{5}}$

$x = -3+\sqrt{5}$
and
$x = -3-\sqrt{5}$

> Sometimes solving this way will give answers that are complex numbers.

Ex Solve $3x^2 + 81 = 0$

$\Rightarrow 3x^2 = -81$

$\Rightarrow x^2 = -27$

$\Rightarrow x = \pm\sqrt{-27}$

$\Rightarrow x = \pm\sqrt{3 \cdot 9 \cdot i^2}$

$\Rightarrow \boxed{x = \pm 3\sqrt{3}\, i}$

x Solve $2(3x-5)^2 + 32 = 0$

$\Rightarrow 2(3x-5)^2 = -32$

$\Rightarrow (3x-5)^2 = -16$

$\Rightarrow 3x-5 = \pm 4i$

$\Rightarrow 3x = 5 \pm 4i$

$\Rightarrow x = \dfrac{5 \pm 4i}{3} = \dfrac{5}{3} \pm \dfrac{4}{3}i$

EX $x^{2/5} - 3x^{1/5} + 2 = 0$ let $u = x^{1/5}$

$\Rightarrow u^2 - 3u + 2 = 0$

$\Rightarrow (u-1)(u-2) = 0$

$\Rightarrow u = 1$ so $x^{1/5} = 1 \Rightarrow \boxed{x = 1}$

or $u = 2$ so $x^{1/5} = 2 \Rightarrow x = 2^5$

$\Rightarrow \boxed{x = 32}$

check: $1^{2/5} - 3^{1/5} + 2 = 0$ ✓

$(32)^{2/5} - 3(32)^{1/5} + 2 = 2^2 - 3 \cdot 2 + 2 = 4 - 6 + 2 = 0$ ✓

EX surface area S of a basketball is $\frac{900}{\pi}$ square inches

find radius, r.

$A = 4\pi r^2$ so $\frac{900}{\pi} = 4\pi r^2 \Rightarrow \frac{225}{\pi^2} = r^2$

$\Rightarrow r = \pm\sqrt{\frac{225}{\pi^2}}$

$\Rightarrow \boxed{r = \pm \frac{15}{\pi} \approx 4.77 \text{ inches}}$

supplementary Exercises: pp. 517-519

1, 3, 15, 21, 25, 33, 37, 43, 47, 49, 65, 71, 101, 105, 109, 113, 123, 127

▸ Sometimes we come upon equations that aren't actually quadratic equations, but the have the same form

$$au^2 + bu + c = 0$$

where u is any algebraic expression (not necessarily a variable)

These are called equations of quadratic form.

› we can use the quadratic equation tools to solve these equations.

Ex $x^4 - 5x^2 + 6 = 0$ this is $(x^2)^2 - 5(x^2) + 6 = 0$

Define $u = x^2$

Then $u^2 - 5u + 6 = 0$

$\Rightarrow (u-2)(u-3) = 0$

$\Rightarrow u = 2, \ u = 3$

so $x^2 = 2$, $x^2 = 3$

$\Rightarrow \boxed{x = \pm\sqrt{2}}$ $\Rightarrow \boxed{x = \pm\sqrt{3}}$

Ex $x - \sqrt{x} - 6 = 0$ let $u = \sqrt{x}$

then $u^2 - u - 6 = 0$

$\Rightarrow (u-3)(u+2) = 0$

$\Rightarrow u = 3, \ u = -2$

so $\sqrt{x} = 3$ $\sqrt{x} = -2$ this can't happen

$\Rightarrow \boxed{x = 9}$ $x = 4$

Check: $9 - \sqrt{9} - 6 = 0$ ✓

$4 - \sqrt{4} - 6 \neq 0$ nope