7.5 Radical Equations and Applications

→ The general idea behind solving radical equations is to isolate the radical on one side of the equal sign & square (cube, etc.) both sides to eliminate the radical.

Ex Solve

$\sqrt{x} - 10 = 0 \Rightarrow \sqrt{x} = 10 \Rightarrow (\sqrt{x})^2 = (10)^2$

$\Rightarrow \boxed{x = 100}$

check: $\sqrt{100} - 10 = 10 - 10 = 0$ ✓

Always check!

Ex $\sqrt{8x} = 6$

$\Rightarrow 8x = 36$

$\Rightarrow x = \frac{36}{8}$

$\Rightarrow \boxed{x = \frac{9}{2}}$

check: $\sqrt{8 \cdot \frac{9}{2}} = \sqrt{\frac{72}{2}} = \sqrt{36} = 6$ ✓ ok

Ex $\sqrt{9-2x} + 9 = 0$

$\Rightarrow \sqrt{9-2x} = -9$ → we see right here that there is no solution

$\Rightarrow 9 - 2x = 81$

$\Rightarrow -2x = 72$

$\Rightarrow x = -36$

check: $\sqrt{9-2(-36)} + 9 = \sqrt{9+72} + 9$

$= \sqrt{81} + 9$

$= 9 + 9$

$= 18 \neq 0$

No solution

→ What if there are 2 radicals?

Note: $\left(\sqrt{3t+1} - \sqrt{t+15}\right)^2 = 3t+1 - 2\sqrt{(3t+1)(t+15)} + t+15$
$= 4t+16 - 2\sqrt{(3t+1)(t+15)}$
→ squaring both sides right away doesn't eliminate the radicals!

Ex $\sqrt{3t+1} - \sqrt{t+15} = 0$

$\Rightarrow \sqrt{3t+1} = \sqrt{t+15}$ → isolate one of the radicals

$\Rightarrow 3t+1 = t+15$

$\Rightarrow 2t = 14$

$\Rightarrow t = 7$

Check: $\sqrt{3(7)+1} - \sqrt{7+15}$
$= \sqrt{22} - \sqrt{22} = 0$ ok ✓

Ex $\sqrt[4]{2x} + \sqrt[4]{x+3} = 0$

$\Rightarrow \sqrt[4]{2x} = -\sqrt[4]{x+3}$ → isolate 1 radical

$\Rightarrow 2x = x+3$ → raise both sides to the 4th power

$\Rightarrow x = 3$

check: $\sqrt[4]{2\cdot 3} + \sqrt[4]{3+3} = \sqrt[4]{6} + \sqrt[4]{6} = 2\sqrt[4]{6} \neq 0$

No solution

Ex $\sqrt{x} + \sqrt{x+2} = 2$

$\Rightarrow \sqrt{x} = 2 - \sqrt{x+2}$ → isolate 1 radical

$\Rightarrow x = \left(2 - \sqrt{x+2}\right)^2$ → square both sides

$\Rightarrow x = \left(2-\sqrt{x+2}\right)\left(2-\sqrt{x+2}\right)$

$\Rightarrow x = 4 - 2\sqrt{x+2} - 2\sqrt{x+2} + x+2$

$\Rightarrow x = x+6 - 4\sqrt{x+2}$

$\Rightarrow -6 = -4\sqrt{x+2}$ → isolate remaining radical

$\Rightarrow 36 = 16(x+2)$ → square

$\Rightarrow 36 = 16x+32 \Rightarrow 4 = 16x \Rightarrow \frac{4}{16} = x \Rightarrow \boxed{x = \frac{1}{4}}$

Check: $\sqrt{\frac{1}{4}} + \sqrt{\frac{1}{4}+2}$
$= \sqrt{\frac{1}{4}} + \sqrt{\frac{1}{4}+\frac{8}{4}}$
$= \sqrt{\frac{1}{4}} + \sqrt{\frac{9}{4}}$
$= \frac{\sqrt{1}}{\sqrt{4}} + \frac{\sqrt{9}}{\sqrt{4}}$
$= \frac{1}{2} + \frac{3}{2} = \frac{4}{2} = 2$ ✓ ok

EX $\sqrt{x+6}+3=\sqrt{x+9}$ → isolate 1 radical

$\Rightarrow (\sqrt{x+6}+3)^2 = x+9$ → square both sides

$\Rightarrow (\sqrt{x+6}+3)(\sqrt{x+6}+3) = x+9$

$\Rightarrow x+6+3\sqrt{x+6}+3\sqrt{x+6}+9 = x+9$

$\rightarrow x+15+6\sqrt{x+6} = x+9$

$\Rightarrow 6\sqrt{x+6} = -6$

$\Rightarrow \sqrt{x+6} = -1$ → isolate remaining radical

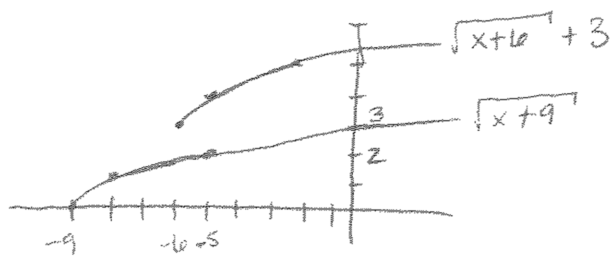
$\Rightarrow x+6=1$ → square

$\Rightarrow x=-5$

check: $\sqrt{-5+6}+3 = \sqrt{1}+3 = 1+3=4$
$\sqrt{-5+9} = \sqrt{4} = 2$
} not equal

$\boxed{\text{No solution}}$

Graphically: $\sqrt{x+6}+3$ is a square root shifted 6 units left + 3 units up
$\sqrt{x+9}$ is a square root shifted 9 units left.



The lines don't cross!

EX $\sqrt{x} = 6-x$

$\Rightarrow x=(6-x)^2$

$\Rightarrow x = 36-12x+x^2$

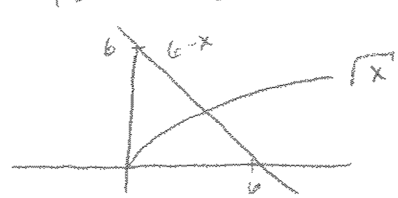
$\Rightarrow x^2-13x+36=0$

$\Rightarrow (x-9)(x-4)=0$

$x=9,4$

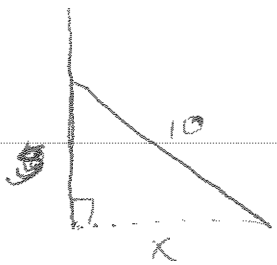
check: $\sqrt{9}=3$; $6-9=-3$ No!
$\sqrt{4}=2$; $6-4=2$ ok ✓

so $\boxed{x=4}$ is the solution



EX 10 foot plank used to brace wall during construction. plank is nailed to wall 5 feet above the floor. what is the slope of the plank?

Draw a picture!



$$\text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{-5}{x}.$$ Need x!

Pythagorean Theorem

$$5^2 + x^2 = 10^2$$
$$25 + x^2 = 100$$
$$x^2 = 75$$
$$x = \sqrt{75}$$
$$x = \sqrt{25 \cdot 3}$$
$$x = 5\sqrt{3} \text{ feet}$$

$$\text{Slope} = \frac{-5}{5\sqrt{3}} = \frac{-1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \boxed{\frac{-\sqrt{3}}{3}}$$

EX Velocity of a free-falling object is given by $V = \sqrt{2gh}$ where g is acceleration due to gravity and h is distance fallen. A coin is dropped from 250 feet above the ground. Find velocity when the coin hits the ground.

→ On earth $g = 32 \text{ feet/second}^2$. Here $h = 250$

$$V = \sqrt{2 \cdot 32 \cdot 250} = \sqrt{16000} = \sqrt{1600 \cdot 10} = 40\sqrt{10} \text{ feet/sec}$$
$$\approx 126.49 \text{ feet/sec}$$
$$\approx 86.24 \text{ mph}$$

Supplementary Exercises

5, 9, 13, 17, 19, 29, 35, 39, 43, 46, 49, 81, 83, 85, 87, 89, 99, 101,