7.4 - Multiplying and Dividing Radical Expressions

→ Multiplying radical expressions is the same idea as multiplying polynomials except that we have to remember the rules of radicals that we saw earlier. Remember, these rules are really just the rules for exponents because radicals are rational exponents

→ unlike dividing polynomials, which involved long division or synthetic division, dividing radical expressions is really just rationalizing the denominator, or getting rid of the radicals in the denominator. We've done simple examples of this previously. Now we'll do a little bit more general examples

<u>Ex</u> multiply remember: $\sqrt[n]{uv} = \sqrt[n]{u} \cdot \sqrt[n]{v}$

a) $\sqrt{6} \cdot \sqrt{18} = \sqrt{6} \cdot \sqrt{3 \cdot 6} = \sqrt{36 \cdot 3} = 6\sqrt{3}$

b) $\sqrt[4]{54} \cdot \sqrt[4]{3} = \sqrt[4]{27 \cdot 2} \cdot \sqrt[4]{3} = \sqrt[4]{81 \cdot 2} = 3\sqrt[4]{2}$ or $3 \cdot 2^{1/4}$

<u>Ex</u> multiply distribute

a) $\sqrt{x}(5 - \sqrt{x}) = 5\sqrt{x} - x$

b) $3\sqrt{5}(\sqrt{5} - \sqrt{2}) = 3\sqrt{25} - 3\sqrt{10} = 15 - 3\sqrt{10}$

c) $\sqrt[3]{9}(\sqrt[3]{3} + 2) = \sqrt[3]{27} + 2\sqrt[3]{9} = 3 + 2\sqrt[3]{9}$ or $3 + 2 \cdot 9^{1/3}$
or $3 + 2 \cdot 3^{2/3}$

<u>Ex</u> multiply foil

a) $(\sqrt{7} + 6)(\sqrt{2} + 6) = \sqrt{14} + 6\sqrt{7} + 6\sqrt{2} + 36$

b) $(10 + \sqrt{2x})^2 = (10 + \sqrt{2x})(10 + \sqrt{2x}) = 100 + 10\sqrt{2x} + 10\sqrt{2x} + 2x$
$= 100 + 20\sqrt{2x} + 2x$

c) $(\sqrt{2}+\sqrt{7})^2 = (\sqrt{2}+\sqrt{7})(\sqrt{2}+\sqrt{7})$
$= 2+\sqrt{14}+\sqrt{14}+7$
$= 9+2\sqrt{14}$

d) $(\sqrt{8}-5)(\sqrt{8}+5) = 8+5\sqrt{8}-5\sqrt{8}-25$
$= -17$

e) $(\sqrt{2}+\sqrt{3})(\sqrt{2}-\sqrt{3}) = 2-\sqrt{6}+\sqrt{6}-3$
$= -1$

Notice in examples d) and e) we started with radicals, but our answer didn't involve radicals.

→ If we have an expression like $5+\sqrt{2}$, The expression $5-\sqrt{2}$ is called the conjugate. The product of the 2 is not a radical.

$$(5+\sqrt{2})(5-\sqrt{2}) = 25-5\sqrt{2}+5\sqrt{2}-4 = 25-4 = 21$$

Ex a) $\sqrt{3}-\sqrt{1}$

recall:
$(a-b)(a+b) = a^2-b^2$

conjugate is $\sqrt{3}+\sqrt{1}$

product is $(\sqrt{3}-\sqrt{1})(\sqrt{3}+\sqrt{1}) = 3+\sqrt{3}-\sqrt{3}-1 = 3-1 = 2$

→ we use the idea of conjugates to rationalize a denominator

Ex Simplify (rationalize the denominator)

$$\frac{8}{\sqrt{7}+3} = \frac{8}{\sqrt{7}+3}\cdot\frac{\sqrt{7}-3}{\sqrt{7}-3} = \frac{8(\sqrt{7}-3)}{7-3\sqrt{7}+3\sqrt{7}-9} = \frac{8(\sqrt{7}-3)}{-2} = -4(\sqrt{7}-3$$
$$= 4(3-\sqrt{7})$$

Ex Simplify

$$\frac{4}{3\sqrt{5}-1} = \frac{4}{3\sqrt{5}-1} \cdot \frac{3\sqrt{5}+1}{3\sqrt{5}+1} = \frac{4(3\sqrt{5}+1)}{9\cdot 5+3\sqrt{5}-3\sqrt{5}-1}$$

$$= \frac{4(3\sqrt{5}+1)}{45-1} = \frac{4(3\sqrt{5}+1)}{44}$$

$$= \frac{3\sqrt{5}+1}{11}$$

Ex $(2\sqrt{t}+1) \div (2\sqrt{t}-1) = \frac{2\sqrt{t}+1}{2\sqrt{t}-1} \cdot \frac{2\sqrt{t}+1}{2\sqrt{t}+1} = \frac{(2\sqrt{t}+1)(2\sqrt{t}+1)}{4t+2\sqrt{t}-2\sqrt{t}-1}$

$$= \frac{4t+2\sqrt{t}+2\sqrt{t}+1}{4t-1} = \frac{4t+4\sqrt{t}+1}{4t-1}$$

Ex $\frac{z}{\sqrt{u+z}-\sqrt{u}} = \frac{z}{\sqrt{u+z}-\sqrt{u}} \cdot \frac{\sqrt{u+z}+\sqrt{u}}{\sqrt{u+z}+\sqrt{u}}$

$$= \frac{z(\sqrt{u+z}+\sqrt{u})}{u+z+\sqrt{u(u+z)}-\sqrt{u(u+z)}-u} = \frac{z(\sqrt{u+z}+\sqrt{u})}{z}$$

$$= \sqrt{u+z}+\sqrt{u} \; ; \; z \neq 0$$

Supplementary Exercises

1, 3, 5, 11, 15, 19, 23, 25, 27, 29, 31, 41, 47, 57, 59, 69, 75, 77, 81, 87, 91, 95, 97