7.2 - Simplifying Radical Expressions

→ Recall that $(xy)^n = x^n y^n$. It works the same for rational exponents: $(xy)^{m/n} = x^{m/n} y^{m/n}$. Since radicals are just rational exponents, the same thing works for radicals. $\sqrt[n]{xy} = \sqrt[n]{x} \cdot \sqrt[n]{y}$

→ we'll use this fact to simplify radicals

Ex Simplify:

a) $\sqrt{72} = \sqrt{36 \cdot 2} = \sqrt{36} \cdot \sqrt{2} = 6 \cdot \sqrt{2} = 6\sqrt{2}$ → look for perfect squares

b) $\sqrt{48} = \sqrt{16 \cdot 3} = \sqrt{16} \cdot \sqrt{3} = 4\sqrt{3}$

c) $\sqrt[3]{24} = \sqrt[3]{8 \cdot 3} = \sqrt[3]{8} \cdot \sqrt[3]{3} = 2\sqrt[3]{3}$ → look for perfect cubes

→ We can do this with variables too, but we have to be careful!

$\sqrt{x^2} \neq x$

notice if $x = 2$ then $\sqrt{4} = 2$ ok ✓

but $x = -2$ then $\sqrt{(-2)^2} = \sqrt{4} \neq -2$

Instead $\sqrt{x^2} = |x|$. Similarly $\sqrt[4]{x^4} = |x|$, $\sqrt[6]{x^6} = |x|$

$\sqrt{x^6} = |x^3|$, $\sqrt[4]{x^{12}} = |x^3|$

But $\sqrt[3]{x^3} = x$

if $x = 2$, $\sqrt[3]{8} = 2$ ok ✓

if $x = -2$, $\sqrt[3]{(-2)^3} = \sqrt[3]{-8} = -2$ ok ✓

similarly $\sqrt[5]{x^5} = x$, $\sqrt[7]{x^7} = x$, $\sqrt[3]{x^4} = x^2$

→ You have to pay attention to the sign to make sure things match up correctly.

EX Simplify

a) $\sqrt{25x^2} = \sqrt{25} \cdot \sqrt{x^2} = 5|x|$

b) $\sqrt{64x^3} = \sqrt{64 \cdot x^2 \cdot x} = \sqrt{64} \cdot \sqrt{x^2} \cdot \sqrt{x} = 8x\sqrt{x}$ → note: we don't need $|x|$ because x has to be greater than or equal to zero anyway.

c) $\sqrt{32x^5} = \sqrt{16 \cdot 2 \cdot x^4 \cdot x} = 4x^2\sqrt{2x}$

d) $\sqrt{125u^4v^6} = \sqrt{25 \cdot 5 \cdot u^4 \cdot v^6} = 5u^2|v^3|\sqrt{5}$ → pay attention to the sign!

e) $\sqrt[3]{32a^5b^6} = \sqrt[3]{8 \cdot 4 \cdot a^3 \cdot a^2 \cdot b^6} = 2ab^2\sqrt[3]{4a^2}$

→ A technicality of working with radicals is that expressions are not considered fully simplified when there are radicals in the denominator of a fraction

→ removing the radicals from the denominator of a fraction is called rationalizing the denominator

Rationalize the denominator:

EX a) $\frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{\sqrt{9}} = \frac{2\sqrt{3}}{3}$

Note: we can't just square the numerator & denominator!

$\frac{1}{3} = \frac{1}{3} \cdot \frac{3}{3} = \frac{3}{9}$, but $\frac{1}{3} \neq \frac{1^2}{3^2} = \frac{1}{9}$

b) $\frac{2}{\sqrt{2}} = \frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}}{\sqrt{4}} = \frac{2\sqrt{2}}{2} = \sqrt{2}$

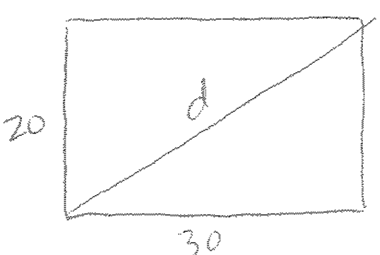
c) $\dfrac{6}{\sqrt{32}} = \dfrac{6}{\sqrt{16 \cdot 2}} = \dfrac{6}{4\sqrt{2}} = \dfrac{3}{2\sqrt{2}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{3\sqrt{2}}{2\sqrt{4}} = \dfrac{3\sqrt{2}}{4}$

↳ it's easier if we give a little thought before we just start multiplying the numerator & denominator by something!

d) $\dfrac{4}{\sqrt[3]{9x^2}} = \dfrac{4 \cdot \sqrt[3]{3x}}{\sqrt[3]{9x^2}\,\sqrt[3]{3x}} = \dfrac{4\sqrt[3]{3x}}{\sqrt[3]{27x^3}} = \dfrac{4\sqrt[3]{3x}}{3x}$

e) $\sqrt{\dfrac{8x}{12y^5}} = \sqrt{\dfrac{2x}{3y^5}} = \dfrac{\sqrt{2x}}{\sqrt{3y^5}} \cdot \dfrac{\sqrt{3y}}{\sqrt{3y}} = \dfrac{\sqrt{6xy}}{\sqrt{9y^6}} = \dfrac{\sqrt{6xy}}{3|y^3|}$

Ex A rectangular room is 20' by 30'. How far is it from one corner to ~~corner~~ the opposite corner?



$d^2 = 20^2 + 30^2$

$d = \sqrt{400 + 900}$

$d = \sqrt{1300}$

$= \sqrt{100 \cdot 13}$

$= 10\sqrt{13} \approx 36.06'$

Supplementary Problems:

1, 3, 5, 7, 19, 23, 29, 31, 43, 45, 55, 57, 61, 67, 71, 73, 77