7.1 Radicals and Rational Exponents

→ We've worked with exponents previously, but always with integer exponents. Now we start working with radicals and see that radicals are really just exponents that are rational numbers but not exponents. Thus, the rules and ideas are similar to what we've already done.

Terminology:

→ $9 = 3^2$ nine equals three squared $\Rightarrow 3 = \sqrt{9}$ 3 is the square root of 9.

→ $-64 = (-4)^3$ -64 equals -4 cubed $\Rightarrow -4 = \sqrt[3]{-64}$ -4 is the cubed root of -64

$16 = 2^4$ 16 equal 2 to the fourth power $\Rightarrow 2 = \sqrt[4]{16}$ 2 is the fourth root of 16

→ we can take an odd numbered root of a negative number but not an even numbered root.

Ex a) $\sqrt{36} = 6$ because $6^2 = 36$ → note: $(-6)^2$ is also 36, but by convention, we take the positive root.
→ we do this for all even roots (square root, 4th root, etc.)

b) $\sqrt[3]{-27} = -3$ because $(-3)^3 = 27$

c) $\sqrt{-16}$ not a real number because you can't square a real number and get 16 as a result

d) $\sqrt[4]{-81}$ is not a real number because you can't raise a number to the fourth power and get -81 as a result. Note: $3^4 = 81$
$(-3)^4 = 81$

→ Remember the rules of exponents

$$a^m \cdot a^n = a^{m+n}$$

Consider:

$$9^{1/2} \cdot 9^{1/2} = 9^{1/2 + 1/2} = 9^1 = 9$$

That means $9^{1/2} = 3$

→ we can write any radical as a rational exponent

Ex → $\sqrt{9} = 9^{1/2} = 3$

→ $\sqrt[3]{-27} = (-27)^{1/3} = 3$

→ $\sqrt[4]{16} = 16^{1/4} = 2$

→ $\sqrt[3]{125} = 125^{1/3} = 5$

What about rational exponents that aren't 1 divided by a number?

Ex → $2^{3/2} = \sqrt[2]{2^3}$ or $\left(\sqrt[2]{2}\right)^3$

→ $3^{4/7} = \sqrt[7]{3^4}$ or $\left(\sqrt[7]{3}\right)^4$

In general $a^{m/n} = \sqrt[n]{a^m}$ or $\left(\sqrt[n]{a}\right)^m$ → numerator power
denominator root

→ we learned rules for integer exponents. The same rules apply to rational exponents

Ex → $3^{1/2} \cdot 3^{2/3} = 3^{1/2 + 2/3} = 3^{3/6 + 4/6} = 3^{7/6}$ or $\sqrt[6]{3^7}$

→ $\frac{2^4}{2^{1/3}} = 2^{4 - 1/3} = 2^{12/3 - 1/3} = 2^{11/3}$

→ $(5^{1/3})^{-1/2} = \frac{1}{(5^{1/3})^{1/2}} = \frac{1}{5^{1/6}}$

Ex $\left(\frac{x^{1/2}\, y^{-2}}{x^{2/3}\, y^{2/5}}\right)^3 = \left(x^{1/2 - 2/3}\, y^{-2 - 2/5}\right)^3$

$= \left(x^{3/6 - 4/6}\, y^{-\frac{10}{5} - \frac{2}{5}}\right)^3$

$= \left(x^{-1/6}\, y^{-12/5}\right)^3$

$= x^{-3/6}\, y^{-36/5}$

$= \frac{1}{x^{1/2}\, y^{36/5}}$

Ex → $8^{4/3} = (8^{1/3})^4 = 2^4 = 16$

→ $-16^{1/2} = -4$ → exponent before negative!

→ $25^{-3/2} = \frac{1}{25^{3/2}} = \frac{1}{(25^{1/2})^3} = \frac{1}{5^3} = \frac{1}{125}$

Domain:

→ If n is odd, the domain of $f(x) = \sqrt[n]{x}$ is all real numbers

→ If n is even, the domain of $f(x) = \sqrt[n]{x}$ is $x \geq 0$.

Ex $\sqrt[3]{-8} = -2$ because $(-2)^3 = -8$. -8 is in the domain of $f(x) = \sqrt[3]{x} = x^{1/3}$

$\sqrt[2]{-16}$ is not defined because we can't square a number & get -16 as an output. -16 is not in the domain of $f(x) = \sqrt{x} = x^{1/2}$

Ex find the domain of $f(x) = x^{3/2}$.
↳ $x^{3/2} = (x^{1/2})^3 = (\sqrt[2]{x})^3$. domain is $x \geq 0$

Ex find the domain of $f(x) = x^{2/3}$
↳ $x^{2/3} = (x^{1/3})^2 = (\sqrt[3]{x})^2$. domain is all real numbers.

→ It's the denominator that matters, not the numerator.

Ex find the domain of $f(x) = \sqrt[4]{3x-2}$
→ don't just need $x \geq 0$. Instead, need $3x - 2 \geq 0 \Rightarrow 3x \geq 2 \Rightarrow \boxed{x \geq 2/3}$

Webwork and radicals

- To enter radicals in webwork, it's usually best to use fractional exponents.
- for $\sqrt{x}$ you can write sqrt(x) or x^(1/2) or x**(1/2)

for $\sqrt[3]{x}$, write x^(1/3) or x**(1/3).

for $\sqrt[3]{x^5}$ write x^(5/3) or x**(5/3)

You <u>must</u> include the parentheses! otherwise, webwork will follow the order of operation and read

$X ** 5/3$ as $\frac{X^5}{3}$ not $X^{5/3}$

<u>Supplementary Problems</u>

1, 3, 5, 15, 17, 19, 25, 27, 53, 55, 57, 77, 81, 83, 89, 91, 93, 117, 119, 151, 153