

## 6.6 Solving Rational Equations

①

→ Solving rational equations has a common theme:

- 1) Multiply by a common denominator to eliminate the fractions
- 2) solve the equation using your favorite method
- 3) check the answer

Ex:  $\frac{x}{4} - \frac{x}{6} = \frac{1}{4}$

→ multiply each term by 12

$$\frac{12x}{4} - \frac{12x}{6} = \frac{12}{4}$$

$$\rightarrow 3x - 2x = 3$$

$$\Rightarrow \boxed{x = 3}$$

check:  $\frac{3}{4} - \frac{3}{6} = \frac{3}{4} - \frac{1}{2} = \frac{3}{4} - \frac{2}{4} = \frac{1}{4}$  ✓

EX  $\frac{x^2}{2} - \frac{3x}{5} = -\frac{1}{10}$

→ multiply each term by 10

$$5x^2 - 6x = -1$$

$$5x^2 - 6x + 1 = 0$$

$$(5x - 1)(x - 1) = 0$$

$$\boxed{x = \frac{1}{5}, 1}$$

check:  $\left(\frac{1}{5}\right)^2 - \frac{3\left(\frac{1}{5}\right)}{5} = \frac{\left(\frac{1}{25}\right)}{2} - \frac{\left(\frac{3}{5}\right)}{5} = \frac{1}{50} - \frac{3}{25} = \frac{1}{50} - \frac{6}{50} = \frac{-5}{50} = -\frac{1}{10}$  ✓

$$\frac{1^2}{2} - \frac{3}{5} = \frac{5}{10} - \frac{6}{10} = -\frac{1}{10}$$
 ✓

→ What about with variables in the denominator?

②

↳ same process, but be sure to check answers!

EX  $\frac{-6}{u+3} = \frac{2}{3}$

multiply each term by  $3(u+3)$

$$-18 = 2(u+3)$$

$$\Rightarrow -9 = u+3$$

$$\Rightarrow \boxed{-12 = u}$$

check:  $\frac{-6}{-9} = \frac{2}{3}$  ✓

EX  $\frac{10}{x(x-2)} + \frac{4}{x} = \frac{5}{x-2}$

multiply each term by  $x(x-2)$

$$10 + 4(x-2) = 5x$$

$$10 + 4x - 8 = 5x$$

$$2 = x$$

check:  $\frac{x}{2(2-2)} + \frac{4}{2}$  uh oh!

The solution we found doesn't work! So there is

$\boxed{\text{no solution}}$

Ex Find a number such that the sum of the number and its reciprocal is  $\frac{37}{6}$

(3)

→ need a variable for the number:  $x$

↳ if the number is  $x$ , its reciprocal is  $\frac{1}{x}$

Then

$$x + \frac{1}{x} = \frac{37}{6}$$

→ multiply each term by  $6x$

$$6x^2 + 6 = 37x$$

$$\Rightarrow 6x^2 - 37x + 6 = 0$$

need to factor this!

factors of 6:	negative factors of 6	possible combos	inner + outer
1, 6	-1, -6	$(x-1)(6x-6)$	$-6x - 6x$
2, 3	-2, -3	$(x-2)(6x-3)$	$-3x - 12x$
3, 2		$(2x-1)(3x-6)$	$-12x - 3x$
6, 1		$(2x-2)(3x-3)$	$-6x - 6x$
		$(3x-1)(2x-6)$	$-18x - 2x$
		$(3x-2)(2x-3)$	$-9x - 4x$
		$(6x-1)(x-6)$	$-36x - x$ ✓ ok!

$$\Rightarrow (6x-1)(x-6) = 0$$

$$\Rightarrow x = \frac{1}{6}, 6$$

If you add  $6 + \frac{1}{6}$  you get  $\frac{37}{6}$

EX Roofer requires 15 hours to ~~shingle~~ a roof. Apprentice requires 21 hours. How long does it take to shingle a roof if they ~~work~~ work together?

Roofer rate:  $\frac{1}{15}$  roof/hour  
 Apprentice rate:  $\frac{1}{21}$  roof/hour

in  $t$  hours, roofer shingles  $\frac{t}{15}$  roofs  
 and apprentice shingles  $\frac{t}{21}$  roofs

Together they shingle  $\frac{t}{15} + \frac{t}{21}$  roofs

Find time,  $t$ , it takes to shingle 1 roof.

$$\frac{t}{15} + \frac{t}{21} = 1 \quad \text{multiply by } 3 \cdot 5 \cdot 7 \quad 7t + 5t = 105$$

$$\Rightarrow 12t = 105 \Rightarrow t = 8.75 \text{ hours}$$

8 hrs 45 min

EX  $1 - \frac{6}{4-x} = \frac{x+2}{x^2-16}$

(4)

$\Rightarrow 1 + \frac{6}{x-4} = \frac{x+2}{(x-4)(x+4)}$

$\rightarrow$  multiply each term by  $(x-4)(x+4)$

$\Rightarrow (x-4)(x+4) + 6(x+4) = x+2$

$\Rightarrow x^2 - 16 + 6x + 24 = x + 2$

$\Rightarrow x^2 + 6x + 8 = x + 2$

$\Rightarrow x^2 + 5x + 6 = 0$

$\Rightarrow (x+2)(x+3) = 0$

$\Rightarrow \boxed{x = -3, -2}$

check:  $1 - \frac{6}{4-(-3)} = 1 - \frac{6}{7} = \frac{1}{7}$   
 $\frac{-3+2}{(-3)^2-16} = \frac{-1}{9-16} = \frac{-1}{-7} = \frac{1}{7}$

$1 - \frac{6}{4-(-2)} = 1 - \frac{6}{6} = 0$   
 $\frac{-2+2}{(-2)^2-16} = 0$

EX One person runs 2 miles per hour faster than a second person. first person runs 5 miles in the same amount of time the second person runs 4 miles. Find speed of each person.

	Rate	time	distance
person 1:	$r+2$	$t$	5
person 2:	$r$	$t$	4

$(r+2)(t) = 5$

We want to solve for  $r$ . Use substitution!

$rt = 4 \rightarrow t = \frac{4}{r}$

$\Rightarrow$  so  $(r+2)\left(\frac{4}{r}\right) = 5 \Rightarrow \frac{4(r+2)}{r} = 5$

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Then  $4r + 8 = 5r$   
 $\Rightarrow \boxed{8 = r}$

5

2nd person runs 8 miles per hour

1st person runs 10 miles per hour

Supplementary Problems: pp. 424-426

5, 9, 11, 21, 31, 33, 43, 45, 53, 61, 63, 65, 83, 85, 87

One more example:

$$\frac{2x}{3} = \frac{1 + \frac{2}{x}}{1 + \frac{1}{x}}$$

$$\Rightarrow \frac{2x}{3} = \frac{1 + \frac{2}{x}}{1 + \frac{1}{x}} \cdot \frac{\frac{1}{x}}{\frac{1}{x}}$$

$$\Rightarrow \frac{2x}{3} = \frac{x+2}{x+1} \quad \rightarrow \text{multiply each term by } 3(x+1)$$

$$\Rightarrow 2x(x+1) = 3(x+2)$$

$$\Rightarrow 2x^2 + 2x = 3x + 6$$

$$\Rightarrow 2x^2 - x - 6 = 0$$

$$\Rightarrow (2x+3)(x-2) = 0$$

$$\Rightarrow \boxed{x = -\frac{3}{2}, 2}$$

Check answers!  $\rightarrow$  these ones work, but you should always check!