6.4 - Complex Fractions

→ Complex fractions are fractions with fractions in the numerator and denominator

→ Working with complex fractions is really the same thing as dividing rational expressions, so we've already done things like this

→ Remember when we divide fractions, we really want to turn the problem into a multiplication problem by multiplying by the reciprocal of the denominator

Ex $\dfrac{\frac{x+2}{3}}{\frac{x-2}{x}}$ This is $\dfrac{x+2}{3} \div \dfrac{x-2}{x}$

$= \dfrac{x+2}{3} \cdot \dfrac{x}{x-2} = \dfrac{x(x+2)}{3(x-2)}$; $x \neq 0$

Ex $\dfrac{\frac{5x}{x+7}}{\frac{10}{x^2+8x+7}} = \dfrac{\frac{5x}{x+7}}{\frac{10}{(x+1)(x+7)}} = \dfrac{5x}{x+7} \cdot \dfrac{(x+1)(x+7)}{10} = \dfrac{x(x+1)}{2}$ $x \neq -7, -1$

We did a lot of these in section 6.2, so let's look at a different kind of problem.

Ex $\dfrac{x}{\left(\frac{3}{x}+2\right)} = \dfrac{x}{\left(\frac{3}{x}+\frac{2x}{x}\right)} = \dfrac{x}{\left(\frac{3+2x}{x}\right)} = \dfrac{x}{1} \cdot \dfrac{x}{3+2x} = \dfrac{x^2}{3+2x}$, $x \neq 0$

(think of this as $\frac{x}{1}$)

↳ need a common denominator

Ex $\dfrac{\left(1+\frac{4}{y}\right)}{y} = \dfrac{\left(\frac{y}{y}+\frac{4}{y}\right)}{y} = \dfrac{\left(\frac{y+4}{y}\right)}{y} = \dfrac{y+4}{y} \cdot \dfrac{1}{y} = \dfrac{y+4}{y^2}$

Ex $\dfrac{\frac{2}{x+5}}{\frac{2}{x+5}+\frac{1}{4x+20}} = \dfrac{\frac{2}{x+5}}{\frac{2}{x+5}+\frac{1}{4(x+5)}} = \dfrac{\frac{2}{x+5}}{\frac{8}{4(x+5)}+\frac{1}{4(x+5)}}$

$= \dfrac{\frac{2}{x+5}}{\frac{9}{4(x+5)}}$

$= \dfrac{2}{x+5} \cdot \dfrac{4(x+5)}{9}$

$= \dfrac{8}{9}\,; \quad x \neq -5$

Ex $\dfrac{x^{-1}+y^{-1}}{x^{-2}-y^{-2}} = \dfrac{\frac{1}{x}+\frac{1}{y}}{\frac{1}{x^2}-\frac{1}{y^2}} = \dfrac{\frac{y}{xy}+\frac{x}{xy}}{\frac{y^2}{x^2y^2}-\frac{x^2}{x^2y^2}} = \dfrac{\frac{x+y}{xy}}{\frac{y^2-x^2}{x^2y^2}}$

$= \dfrac{x+y}{xy} \cdot \dfrac{x^2y^2}{y^2-x^2}$

$= \dfrac{x+y}{xy} \cdot \dfrac{x^2y^2}{(y-x)(y+x)}$

$= \dfrac{xy}{y-x}\,; \quad x \neq 0,\ y \neq 0,\ x \neq -y$

→ Remember that hard problem from the last homework? We now have the tools to solve it.

EX $$\frac{-3x^{-6}-3y^{-6}}{(x^2+y^2)y^{-4}} = \frac{\frac{-3}{x^6}-\frac{3}{y^6}}{\frac{x^2+y^2}{y^4}} = \frac{\frac{-3y^6}{x^6y^6}-\frac{3x^6}{x^6y^6}}{\frac{x^2+y^2}{y^4}}$$

$$= \frac{\frac{-3y^6-3x^6}{x^6y^6}}{\frac{x^2+y^2}{y^4}}$$

$$= \frac{-3(y^6-x^6)}{x^6y^6}\cdot\frac{y^4}{x^2+y^2}$$

Sum of cubes:

$u^3+v^3=(u+v)(u^2-uv+v^2)$

and $y^6+x^6=(y^2)^3+(x^2)^3$

$$= \frac{-3(y^6+x^6)}{x^6y^2(x^2+y^2)}$$

$$= \frac{-3(y^2+x^2)(y^4-y^2x^2+x^4)}{x^6y^2(x^2+y^2)}$$

$$= \frac{-3(y^4-y^2x^2+x^4)}{x^6y^2}$$

→ Let's look at another way to solve problems like this. It's the same thing we've been doing, but it's a bit shorter

→ We'll think about complex fractions with only numbers first.

Consider $\dfrac{\frac{1}{3}+\frac{1}{4}}{\frac{1}{3}+\frac{1}{2}} = \dfrac{\frac{4}{12}+\frac{3}{12}}{\frac{2}{6}+\frac{3}{6}} = \dfrac{\frac{7}{12}}{\frac{5}{6}} = \frac{7}{12}\cdot\frac{6}{5}$

$= \frac{7}{10}$

→ This is what we've been doing.

Another approach is to find a common denominator for all the fractions in the problem & multiply the numerator & denominator by that.

$$\frac{\frac{1}{3}+\frac{1}{4}}{\frac{1}{3}+\frac{1}{2}} \cdot \frac{\frac{12}{1}}{\frac{12}{1}}$$

↗ we're just multiplying by 1 so it's legal.

$$= \frac{\frac{12}{3}+\frac{12}{4}}{\frac{12}{3}+\frac{12}{2}} = \frac{4+3}{4+6} = \frac{7}{10}$$

→ we can do the same thing with rational expressions

CD of all fractions: $2x(x+5)(x-5)$

Ex $\dfrac{\frac{1}{2x}-\frac{6}{x+5}}{\frac{x}{x-5}+\frac{1}{x}}$

↗ don't multiply!

$$= \frac{\frac{1}{2x}-\frac{6}{x+5}}{\frac{x}{x-5}+\frac{1}{x}} \cdot \frac{\frac{2x(x+5)(x-5)}{1}}{\frac{2x(x+5)(x-5)}{1}} = \frac{(x+5)(x-5)-6(2x)(x-5)}{x\cdot 2x(x+5)+2(x+5)(x-5)}$$

$$= \frac{(x-5)(x+5-12x)}{2(x+5)(x^2+x-5)}$$

$$= \frac{(x-5)(5-11x)}{2(x+5)(x^2+x-5)}; \quad x\neq 0, 5$$

or $$-\frac{(x-5)(11x-5)}{2(x+5)(x^2+x-5)}; \quad x\neq 0, 5$$

EX $\dfrac{\left(x - \dfrac{6y^2}{x-y}\right)}{x-3y}$

Common denominator: $x-y$

remember this has a denominator of 1

$$= \frac{x - \dfrac{6y^2}{x-y}}{x-3y} \cdot \frac{\dfrac{x-y}{1}}{\dfrac{x-y}{1}}$$

$$= \frac{x(x-y) - 6y^2}{(x-3y)(x-y)}$$

$$= \frac{x^2 - xy - 6y^2}{(x-3y)(x-y)}$$

$$= \frac{(x-3y)(x+2y)}{(x-3y)(x-y)}$$

$$= \frac{x+2y}{x-y} \; ; \; x \neq 3y$$

Supplementary Problems: pp. 404-407

3, 5, 7, 11, 13, 23, 27, 33, 39, 45, 47, 49, 55, 57, 61 (hard)