

6.3 Adding and Subtracting Rational Expressions

→ Just like fractions, we need a common denominator to add or subtract rational expressions.

→ when we already have a common denominator, it's easy

$$\begin{aligned}\text{EX} \quad \frac{16+z}{5z} - \frac{11-z}{5z} &= \frac{16+z - (11-z)}{5z} \\ &= \frac{16+z-11+z}{5z} \\ &= \frac{2z+5}{5z}\end{aligned}$$

$$\text{EX} \quad \frac{d}{d^2-36} - \frac{6}{d^2-36} = \frac{d-6}{d^2-36} = \frac{d-6}{(d-6)(d+6)} = \frac{1}{d+6} ; d \neq 6$$

$$\begin{aligned}\text{EX} \quad \frac{x^2-4x}{x-3} + \frac{10-4x}{x-3} - \frac{x-8}{x-3} &= \frac{x^2-4x+10-4x-x+8}{x-3} \\ &= \frac{x^2-9x+18}{x-3} \\ &= \frac{\cancel{(x-3)}(x-6)}{\cancel{x-3}} = x-6 ; x \neq 3\end{aligned}$$

→ Things are much more difficult when we don't have a common denominator.

$$\text{Ex } \frac{5}{4x} - \frac{3}{5}$$

→ for a common denominator, we need to find the least common multiple of $4x$ and 5 .

$$= \frac{5 \cdot 5}{4x \cdot 5} - \frac{3 \cdot 4x}{5 \cdot 4x}$$

→ LCM = $20x$, so that is our common denominator

$$= \frac{25}{20x} - \frac{12x}{20x}$$

$$= \frac{25 - 12x}{20x}$$

$$\text{Ex } \frac{1}{6u^2} - \frac{2}{9u}$$

Common denominator: $18u^2$

$$= \frac{1 \cdot 3}{6u^2 \cdot 3} - \frac{2 \cdot 2u}{9u \cdot 2u}$$

$$= \frac{3}{18u^2} - \frac{4u}{18u^2}$$

$$= \frac{3 - 4u}{18u^2}$$

→ things can get trickier with more complicated denominators

$$\text{Ex } \frac{x}{x+3} - \frac{5}{x-2}$$

Common denominator: $(x+3)(x-2)$

$$= \frac{x(x-2)}{(x+3)(x-2)} - \frac{5(x+3)}{(x+3)(x-2)}$$

$$= \frac{x^2 - 2x - 5x - 15}{(x+3)(x-2)} = \frac{x^2 - 7x - 15}{(x+3)(x-2)}$$

→ numerator not factorable, so this is our final answer

$$\text{Ex } \frac{6x}{x^2-4} + \frac{3}{2-x}$$

$$= \frac{6x}{(x-2)(x+2)} + \frac{3}{2-x}$$

→ factor first! It makes it easier to find a common denominator.

③

$$= \frac{6x}{(x-2)(x+2)} - \frac{3}{x-2}$$

→ common denominator: $(x-2)(x+2)$

$$= \frac{6x}{(x-2)(x+2)} - \frac{3(x+2)}{(x-2)(x+2)}$$

$$= \frac{6x - 3x - 6}{(x-2)(x+2)}$$

$$= \frac{3x - 6}{(x-2)(x+2)}$$

$$= \frac{3(x-2)}{(x-2)(x+2)}$$

$$= \frac{3}{x+2}; x \neq 2$$

$$\text{Ex } \frac{1}{x-y} - \frac{3}{x+y} + \frac{3x-y}{x^2-y^2}$$

→ factor!

$$= \frac{1}{(x-y)} - \frac{3}{(x+y)} + \frac{3x-y}{(x-y)(x+y)}$$

→ Common Denominator: $(x-y)(x+y)$

$$= \frac{1(x+y)}{(x-y)(x+y)} - \frac{3(x-y)}{(x-y)(x+y)} + \frac{3x-y}{(x-y)(x+y)}$$

↳

$$= \frac{x+y - 3x + 3y + 3x - y}{(x-y)(x+y)}$$

$$= \frac{x+3y}{(x-y)(x+y)}$$

Ex $\frac{-2x-10}{x^2+8x+15} + \frac{2}{x+3} + \frac{x}{x+5}$

→ factor denominator

$$= \frac{-2x-10}{(x+3)(x+5)} + \frac{2}{x+3} + \frac{x}{x+5}$$

CD: (x+3)(x+5)

$$= \frac{-2x-10}{(x+3)(x+5)} + \frac{2(x+5)}{(x+3)(x+5)} + \frac{x(x+3)}{(x+3)(x+5)}$$

$$= \frac{-2x-10 + 2x+10 + x^2+3x}{(x+3)(x+5)}$$

$$= \frac{x^2+3x}{(x+3)(x+5)}$$

$$= \frac{x(x+3)}{(x+3)(x+5)}$$

$$= \frac{x}{x+5}; x \neq -3$$

Supplementary Problems: pp. 396-399

1, 3, 9, 13, 17, 51, 53, 55, 67, 73, 71, 75, 77, 81