

6.2 - Multiplying and Dividing Rational Expressions

(1)

→ Multiplying/dividing rational expressions is just like multiplying or dividing fractions. We multiply the numerators and denominators and for division, we multiply by the reciprocal of the expression we're dividing by.

→ What we do here, is pretty much the same as section 6.1. We multiply and then simplify by cancelling common terms multiplying both the numerator and denominator.

$$\text{EX} \quad \frac{3x^4}{7x} \cdot \frac{8x^2}{9} = \frac{8x^5}{21}, x \neq 0$$

→ Our original expression had a domain of $x \neq 0$, so we restrict the domain in our final answer to be the same as our original domain.

$$\text{EX} \quad \frac{18x^4}{1} \cdot \frac{4}{18x} = \frac{24x^3}{5}; x \neq 0$$

$$\begin{aligned} \text{EX} \quad \frac{(x-2y)^2}{x+2y} \cdot \frac{x^2+7xy+10y^2}{x^2-4y^2} &= \frac{(x-2y)^2}{(x+2y)} \cdot \frac{(x+2y)(x+5y)}{(x-2y)(x+2y)} \\ &= \frac{(x-2y)(x+5y)}{x+2y}; x \neq 2y \end{aligned}$$

Ex $\frac{x+5}{x-5} \cdot \frac{2x^2-9x-5}{3x^2+x-2} \cdot \frac{x^2-1}{x^2+7x+10}$

= $\frac{x+5}{x-5} \cdot \frac{(2x+1)(x-5)}{(3x-2)(x+1)} \cdot \frac{(x-1)(x+1)}{(x+5)(x+2)}$

= $\frac{(2x+1)(x-1)}{(3x-2)(x+2)} ; x \neq -5, -1, 5$

Let's do a couple of examples with division

Ex $\frac{x+3}{4} \div \frac{x-2}{x} \rightarrow$ we can write this as $\frac{\left(\frac{x+3}{4}\right)}{\left(\frac{x-2}{x}\right)}$

= $\frac{x+3}{4} \cdot \frac{x}{x-2}$

= $\frac{x(x+3)}{4(x-2)} ; x \neq 0$

↳ we have three fractions here, and we don't want to divide by zero in any of them

Top fraction: $\frac{x+3}{4} \rightarrow$ we'll never divide by zero

bottom fraction: $\frac{x-2}{x} \rightarrow$ need $x \neq 0$

Big fraction: $\frac{\left(\frac{x+3}{4}\right)}{\left(\frac{x-2}{x}\right)} \rightarrow$ we divide by zero if $x=2$, so need $x \neq 2$

Thus the domain is $x \neq 0, 2$.

\rightarrow In general can't let numerator or denominator equal zero in the thing we're dividing by.

Ex $\frac{x^2-9}{5(x+2)} \div \frac{x+3}{5(x^2-4)}$

$= \frac{(x-3)(x+3)}{5(x+2)} \div \frac{x+3}{5(x-2)(x+2)}$

Domain: $x \neq -3, -2, 2$

$= \frac{(x-3)(x+3)}{5(x+2)} \cdot \frac{5(x-2)(x+2)}{x+3}$

$= (x-2)(x-3); x \neq -3, -2, 2$

Ex $\left(\frac{x^2+6x+9}{x^2} \cdot \frac{2x+1}{x^2-9} \right) \div \frac{4x^2+4x+1}{x^2-3x}$

$= \left(\frac{(x+3)(x+3)}{x^2} \cdot \frac{2x+1}{(x-3)(x+3)} \right) \div \frac{(2x+1)(2x+1)}{x(x-3)}$

Domain: $x \neq 0, 3, -1/2$

$= \frac{\cancel{(x+3)}(x+3)}{x^2} \cdot \frac{2x+1}{(x-3)\cancel{(x+3)}} \cdot \frac{x\cancel{(x-3)}}{(2x+1)(2x+1)}$

$= \frac{x+3}{x(2x+1)}; x \neq \pm 3$

Supplementary Problems: pp. 387-390

9, 11, 15, 21, 27, 29, 37, 41, 45, 49, 53, 57