# 6.1 - Rational Expressions and Functions



→ A rational expression is just like a fraction except now the numerator and denominator are polynomials instead of just numbers.

→ In terms of adding/subtracting, we do the same thing we did with fractions, and we must find a common denominator. We'll do this in section 6.3

→ To multiply, we just multiply numerators & denominators like we did with fractions, & ~~we have to~~ to divide, we have to multiply by the reciprocal like we did with fractions. This is section 6.2

→ In this section, we'll look at finding domains & simplifying rational expressions.

Examples:

| rational expression | Domain → All numbers that don't make us divide by zero |
|---|---|
| $\frac{3}{x+4}$ | All $\mathbb{R}$ except $x \neq 4$ |
| $\frac{2x}{x^2-4x+4} = \frac{2x}{(x-2)^2}$ | All $\mathbb{R}$, but $x \neq 2$ |
| $\frac{x^2-5x}{x^2+2x-3} = \frac{x(x-5)}{(x+3)(x-1)}$ | $x \neq -3, 1$ |

→ The domain is the set of all $x$ for which the denominator is not equal to zero.

→ Simplifying rational expressions means cancelling out common terms multiplying both the numerator and denominator

EX $\dfrac{4}{2x+4}$ Domain: $2x+4=0 \Rightarrow 2x=-4 \Rightarrow x=-2$

so domain is $x \neq -2$

$= \dfrac{2 \cdot 2}{2(x+2)}$

$= \dfrac{2}{x+2}$

→ Note: we can cancel out terms that are multiplied, not terms that are added. So we can't cancel the 2 in the numerator with the 2 added in the denominator.

→ we can check the answer by substituting any number in the domain. Try $x=1$. Then $\dfrac{4}{2(1)+4} = \dfrac{4}{6} = \dfrac{2}{3}$ and $\dfrac{2}{1+2} = \dfrac{2}{3}$ so it checks out.

EX simplify $\dfrac{2x^3-6x}{6x^2}$ Domain: $x \neq 0$

→ we almost always start out by factoring the numerator & denominator.

$$\frac{2x^3-6x}{6x^2} = \frac{2x(x^2-3)}{2x \cdot 3x} = \frac{x^2-3}{3x}$$

**Ex** $\dfrac{x^2+4x+4}{2x^2+4x}$

$= \dfrac{(x+2)(x+2)}{2x(x+2)}$

$= \dfrac{x+2}{2x}$ ; $x \neq -2$

Domain at start: $2x(x+2)=0$

$\Rightarrow x=0, x=-2$

so domain is $x \neq 0, -2$

Domain at end: $2x=0 \Rightarrow x=0$

so $x \neq 0$.

→ Technically, for things to be equal, they must have the same domain. So we restrict the domain of our final result to make it the same as the starting point.

Check: $x=1$ $\quad \dfrac{1+4+4}{2+4} = \dfrac{9}{6} = \dfrac{3}{2}$

$\dfrac{1+2}{2} = \dfrac{3}{2}$ ✓

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**Ex** $\dfrac{x^2-4}{2x-4}$

$= \dfrac{(x-2)(x+2)}{2(x-2)}$

$= \dfrac{x+2}{2}$ ; $x \neq 2$

Domain at start: $x \neq 2$

Domain at end: $\mathbb{R}$

For the expressions to be equal, we must restrict the domain of the ending expression to make it the same as the beginning expression.

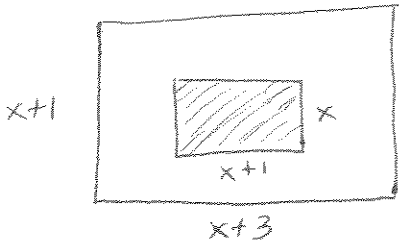
Check: $\dfrac{1-4}{2-4} = \dfrac{-3}{-2} = \dfrac{3}{2}$

$\dfrac{1+2}{2} = \dfrac{3}{2}$ ✓

→ the next example is a bit trickier.

**Ex** $\dfrac{x^2+2x-15}{9-3x} = \dfrac{(x+5)(x-3)}{3(3-x)} = \dfrac{(x+5)(x-3)}{-3(x-3)} = -\dfrac{x+5}{3}$ ; $x \neq 3$

**Ex** You toss a marble randomly into the box shown below. What is the probability that you throw the marble into the shaded region?




$$\text{Probability} = \frac{\text{area of shaded region}}{\text{Total area of large region}}$$

$$= \frac{x(x+1)}{(x+1)(x+3)}$$

$$= \frac{x}{x+1}$$

Domain in context of problem? $x > 0$.

**Ex** 2 pools, one circular and one rectangular. Rectangular pool's width is three times its depth. Length is 6 feet more than width. Circular pool has a diameter that is twice the width of the rectangular pool. Depth is 2 feet deeper than rectangular pool. Find ratio of volume of circular pool to volume of rectangular pool.

Rectangular pool: volume $= lwd_R$

Circular pool : volume $= \pi r^2 d_C$

Rectangular pool: let $d_R$ = depth

$w = 3d_R$

$l = w + 6 = 3d_R + 6$

Volume $= (d_R)(3d_R)(3d_R+6)$

Circular pool

diameter $= 2w \Rightarrow r = w$ (since diameter = 2 · radius)

$\Rightarrow r = 3d_R$

$d_C = d_R + 2$

volume $= \pi (3d_r)^2 (d_R + 2)$

So then the ratio of the volumes is

$$\frac{\pi(3d)^2(d+2)}{d(3d)(3d+6)} = \frac{\pi(3d)^2(d+2)}{d(3d)(3)(d+2)}$$

$$= \frac{\pi(3d)^2(d+2)}{(3d)(3d)(d+2)}$$

$$= \pi$$

What is the domain in the context of the problem?

, want a positive depth, so $d>0$

Supplementary Problems

1, 3, 5, 13, 15, 21, 43, 45, 47, 57, 61, 63, 73, 75, 85,