5.6 - Solving Polynomial Equations By Factoring

→ Suppose we want to solve the equation

$$x^2 - 2x - 8 = 0$$

→ Notice that we can factor the left hand side to get

$$(x-4)(x+2) = 0$$

→ Now remember that if we have to numbers, a and b, then the only way that $ab = 0$ is if $a = 0$ or $b = 0$ or $a = b = 0$

→ The same is true above. If

$(x-4)(x+2) = 0$ then $x-4 = 0 \Rightarrow \boxed{x = 4}$

or $x+2) = 0 \Rightarrow \boxed{x = -2}$

→ There are <u>two</u> solutions to the equation, $x = -2, 4$

<u>EX</u> Solve $2x^2 + 5x = 12$

→ we first subtract 12 from both sides to get

$$2x^2 + 5x - 12 = 0$$

→ Now factor the left side.

factors of 2	factors of -12	possible combinations	Inner + outer
1,2	1,-12	$(x+1)(2x-12)$	$-12x+2x$
2,1	-1,12	$(x-1)(2x+12)$	$12x-2x$
	2,-6	$(x+2)(2x-6)$	$-6x+4x$
	-2,6	$(x-2)(2x+6)$	$6x-4x$
	3,-4	$(x+3)(2x-4)$	$-4x+6x$
	-3,4	$(x-3)(2x+4)$	$4x-6x$

Continued...

possible combos	inner + outer
$(2x+1)(x-12)$	$-24x+x$
$(2x-1)(x+12)$	$24x-x$
$(2x+2)(x-6)$	$-12x+2x$
$(2x-2)(x+6)$	$12x-2x$
$(2x+3)(x-4)$	$-8x+5x$
$(2x-3)(x+4)$	$8x-3x$ ✓ Finally found it!

so then

$$2x^2+5x-12=0 \Rightarrow (2x-3)(x+4)=0$$

so then $2x-3=0 \Rightarrow 2x=3 \Rightarrow \boxed{x=\frac{3}{2}}$ or $x+4=0 \Rightarrow \boxed{x=-4}$

2 solutions: $x=-4, \frac{3}{2}$

Note: we <u>can't</u> say $2x^2+5x=12 \Rightarrow x(2x+5)=12$

so $x=12$ or $2x+5=12$

This is <u>NOT</u> the solution.

If we say $ab=6$, we <u>cannot</u> infer that $a=6$ or $b=6$

<u>EX</u> Solve $x^2+2x+9=8x$

First, subtract 8x from both sides!

$x^2-6x+9=0$ → this is a perfect square!

$(x-3)^2=0$

$(x-3)(x-3)=0$

so $(x-3)=0 \Rightarrow \boxed{x=3}$ or $x-3=0 \Rightarrow x=3$

There is one solution repeated 2 times.

Ex The sum of a positive number and its square is 20. Find the number.

→ Give the number a variable, x

$$x + x^2 = 20$$

→ we can solve by factoring

$$x^2 + x - 20 = 0$$

$\Rightarrow (x+5)(x-4) = 0 \quad \Rightarrow \quad x+5=0 \Rightarrow x=-5$

or $\quad x-4=0 \Rightarrow x=4$

→ There are two numbers ~~answers~~, but only 4 is positive, so it is the answer.

Ex Solve $\quad x^3 - 13x^2 + 42x = 0$

Factor out an x. $\quad x(x^2 - 13x + 42) = 0$

so $\boxed{x=0}$ or $\quad x^2 - 13x + 42 = 0$ → we can factor again

$(x-6)(x-7) = 0$

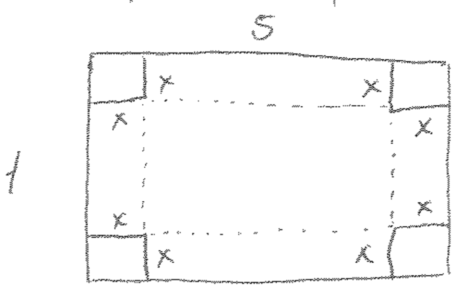
so $\quad x-6=0 \Rightarrow \boxed{x=6}$

or $\quad x-7=0 \Rightarrow \boxed{x=7}$

3 solutions: $x = 0, 6, 7$

Ex Suppose you have a flat piece of card board that is 4 feet wide and 5 feet long. You want to make this into a box by cutting out squares of dimension x from each corner and turning up the sides. what is the volume of the box as a function of x?

→ Draw a picture!



New length: $5-2x$
New width: $4-2x$
height: x

so then $V(x) = (5-2x)(4-2x)x$

→ when is the Volume equal to zero?

$(5-2x)(4-2x)x = 0 \Rightarrow \boxed{x=0}$, $4-2x=0 \Rightarrow -2x=-4 \Rightarrow \boxed{x=2}$

or $5-2x=0 \Rightarrow -2x=-5 \Rightarrow \boxed{x=5/2}$

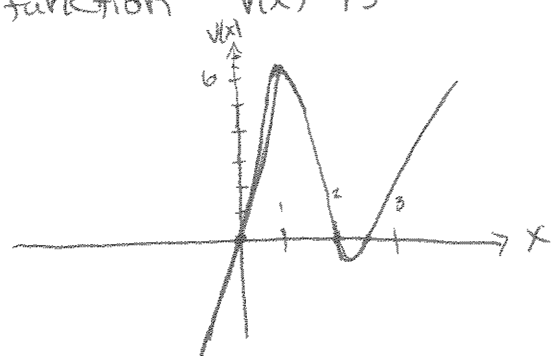
→ what is an appropriate domain for V(x) in the context of the problem?

→ Remember, the domain is the set of possible inputs.

→ we want the volume to be greater than zero. Also ~~Equivalently~~, we want all the lengths in the picture to be positive. This gives the domain as

$$\boxed{0 < x < 2}$$

Graphically, the function V(x) is



Ex Solve $9x^4 - 15x^3 - 9x^2 + 15x = 0$

$\Rightarrow 3x(3x^3 - 5x^2 - 3x + 5) = 0$

$\Rightarrow 3x\left[x^2(3x-5) - (3x-5)\right] = 0$

$\Rightarrow 3x\left[(3x-5)(x^2-1)\right] = 0$

$\Rightarrow 3x(3x-5)(x-1)(x+1) = 0$

so $3x = 0 \Rightarrow \boxed{x=0}$, $3x-5=0 \Rightarrow 3x=5 \Rightarrow \boxed{x = 5/3}$

$x-1=0 \Rightarrow \boxed{x=1}$, $x+1=0 \Rightarrow \boxed{x=-1}$

Supplementary Problems

13, 15, 17, 33, 35, 45, 59, 75, 97, 101, 103, 107