

5.4 Factoring By Grouping and Special forms

①

→ Yesterday we multiplied polynomials together to get a result.

Today we start looking at the reverse process called factoring.

→ the easiest type of factoring is to factor out common monomial factors.

EX factor $6x^3 + 15x$

$$6x^3 + 15x = (3x)(2x^2) + (3x)(5) = 3x(2x^2 + 5)$$

EX $-2x^4 + 6x^3 + 2x = (-2x)(x^3) + (-2x)(-3x^2) + (-2x)(-1)$
 $= -2x(x^3 - 3x^2 - 1)$

EX $-2x^4 + 6x^3 + 2x = 2x(-x^3) + (2x)(3x^2) + (2x)(1)$
 $= 2x(-x^3 + 3x^2 + 1)$

Two ways of writing the same thing.

→ Another relatively easy type of factoring is to recognize special forms.

EX factor $4x^2 - 9$

→ difference of squares

$$4x^2 - 9 = (2x - 3)(2x + 3)$$

EX factor $x^2 - 2x + 1$ \rightarrow perfect square (2).
 $x^2 - 2x + 1 = (x-1)^2$ $(u-v)^2 = u^2 - 2uv + v^2$

EX factor $4x^4 + 12x^2 + 9$ \rightarrow perfect square
 $4x^4 + 12x^2 + 9 = (2x^2 + 3)^2$ $(u+v)^2 = u^2 + 2uv + v^2$

EX factor $8x^2 - 2$ \rightarrow difference of squares
 $8x^2 - 2 = 2(4x^2 - 1)$
 $= 2(2x-1)(2x+1)$

\rightarrow Two other special forms are sums and differences of cubes

$$u^3 + v^3 = (u+v)(u^2 - uv + v^2)$$

$$u^3 - v^3 = (u-v)(u^2 + uv + v^2)$$

\rightarrow difference of cubes

EX factor $24x^4 - 3x = 3x(8x^3 - 1)$ $\begin{matrix} u=2x \\ v=1 \end{matrix}$
 $= 3x(2x-1)(4x^2 + 2x + 1)$

\rightarrow Another method for factoring is factoring by grouping
 \hookrightarrow this method doesn't apply most of the time, but is occasionally useful

Ex $y^2 + 3y + 4y + 12$

$$= y \cdot y + 3 \cdot y + 4 \cdot y + 4 \cdot 3$$

$$= y(y+3) + 4(y+3)$$

$$= (y+3)(y+4)$$

(3)

EX $x^3 + 2x^2 + x + 2$

$$= x^2(x+2) + 1(x+2)$$

$$= (x+2)(x^2+1)$$

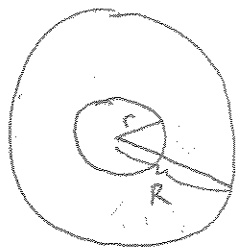
EX $3x^3 + 2x^2 - 3x - 2$

$$= x^2(3x+2) - 1(3x+2)$$

$$= (3x+2)(x^2-1)$$

$$= (3x+2)(x-1)(x+1)$$

EX Find the shaded area below.



Area of big circle: πR^2

Area of small circle: πr^2

$$\text{shaded area} = \pi R^2 - \pi r^2 = \pi (R^2 - r^2)$$

$$= \pi (R-r)(R+r)$$

Supplementary Problems: pp. 334-337

21, 25, 27, 35, 39, 55, 57, 65, 67, 69, 71, 99, 101, 103, 111, 113, 115, 139