5.1 Integer Exponents and Scientific Notation

→ Let's review some terminology

$$a^n = \underbrace{a \cdot a \cdot a \cdots a}_{n \text{ factors}}$$

a is the base
n is the exponent

→ we say a to the power of n

→ Let's try to think about the properties of exponents.

1) Consider

$$(x^2)(x^3) = (x \cdot x)(x \cdot x \cdot x) = x \cdot x \cdot x \cdot x \cdot x = x^5$$

In general $a^n \cdot a^m = a^{n+m}$ → note that we need the same base to do this.

2) $(x \cdot y)^4 = (x \cdot y)(x \cdot y)(x \cdot y)(x \cdot y) = x \cdot y \cdot x \cdot y \cdot x \cdot y \cdot x \cdot y$

$$= x \cdot x \cdot x \cdot x \cdot y \cdot y \cdot y \cdot y$$

$$= x^4 \cdot y^4$$

In general $(ab)^n = a^n b^n$

→ One thing to remember before we proceed:

$a^0 = 1$ for any a (except $a = 0$)

Notice: $x^0 x^m = x^{0+m} = x^m$, so $x^0 = 1$

0^0 is undefined

3) $x^2 \cdot x^{-2} = x^{2-2} = x^0 = 1$

So then $x^{-2} = \frac{1}{x^2}$

In General $a^{-m} = \frac{1}{a^m}$ and $\frac{1}{a^{-m}} = \frac{1}{\left(\frac{1}{a^m}\right)} = 1 \cdot \frac{a^m}{1} = a^m$

4) $\frac{x^5}{x^3} = x^5 \cdot \frac{1}{x^3} = x^5 \cdot x^{-3} = x^{5-3} = x^2$

In general $\frac{a^m}{a^n} = a^{m-n}$

5) $\left(\frac{x}{y}\right)^3 = \left(\frac{x}{y}\right)\left(\frac{x}{y}\right)\left(\frac{x}{y}\right) = \frac{x \cdot x \cdot x}{y \cdot y \cdot y} = \frac{x^3}{y^3}$

In general $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

6) $(x^2)^3 = (x^2)(x^2)(x^2) = (x \cdot x)(x \cdot x)(x \cdot x)$
$= x \cdot x \cdot x \cdot x \cdot x \cdot x = x^6$

In general $(a^m)^n = a^{m \cdot n}$

→ The distance from the earth to the sun is approximately 93 million miles. This is

93,000,000

↳ It's tedious to write this many zeros. We can write

9.3×10^7 instead.

↳ Think of this as taking 9.3 and moving the decimal 7 places to the right.

$$9.3 \times 10^7 = 93 \times 10^6 = 930 \times 10^5 = 9300 \times 10^4 \text{, etc} = 93{,}000{,}000$$

Similarly,

5.23×10^6 is 5,230,000

→ We can do this for really small decimals too

6.2×10^{-5}

↳ move the decimal 5 places to the left

so $6.2 \times 10^{-5} = 0.000062$

Similarly

$$0.0000125 = 1.25 \times 10^{-5} = 0.125 \times 10^{-4} = 12.5 \times 10^{-6}$$

→ On a calculator, the button for scientific notation is sometimes E or EE.

→ to enter 6.02×10^{23}, for example, you can type

$(6.02) \cdot 10^{\wedge}(23)$ or $\underbrace{6.02\,E23}$

↳ this works in WebWork

→ Let's combine some of these thing together

Ex write with positive exponents

a) $\left(\frac{a}{b}\right)^{-2} = \frac{1}{\left(\frac{a}{b}\right)^2} = \frac{1}{\frac{a^2}{b^2}} = 1 \cdot \frac{b^2}{a^2} = \frac{b^2}{a^2} = \left(\frac{b}{a}\right)^2$

b) $(4a^{-2})^{-3} = 4^{-3} \cdot a^{(-2)(-3)} = \frac{1}{4^3} \cdot a^6 = \frac{a^6}{64}$

c) $\frac{2y^{-1}z^{-3}}{4yz^{-3}} = \frac{2 \cdot \frac{1}{y} \cdot \frac{1}{z^3}}{4 \cdot y \cdot \frac{1}{z^3}} = \frac{\frac{2}{yz^3}}{\frac{4y}{z^3}} = \frac{2}{yz^3} \cdot \frac{z^3}{4y} = \frac{2z^3}{4z^3 y \cdot y}$

$= \frac{z^{3-3}}{2y^{1+1}} = \frac{1}{2y^2}$

d) $(u+v^{-2})^{-1} = \frac{1}{u+v^{-2}} = \frac{1}{u+\frac{1}{v^2}} = \frac{1}{\frac{uv^2}{v^2}+\frac{1}{v^2}} = \frac{1}{\frac{uv^2+1}{v^2}}$

$= 1 \cdot \frac{v^2}{uv^2+1} = \frac{v^2}{uv^2+1}$

→ Be careful when you have addition/subtraction!

e) $\left(\frac{a^{-3}}{b^{-3}}\right)\left(\frac{2b}{a}\right)^3 = \left(\frac{a^{-3}}{b^{-3}}\right)\left(\frac{2^3b^3}{a^3}\right) = \frac{8a^{-3}b^3}{b^{-3}a^3} = \frac{8b^{3-(-3)}}{a^{3-(-3)}} = \frac{8b^6}{a^6} = 8\left(\frac{b}{a}\right)^6$

→ Let's move on to a related topic, scientific notation.

Supplementary practice problems: pp. 304-307

1, 3, 5, 9, 11, 13, 15, 21, 23, 25, 27, 41, 47, 49, 55, 59, 65, 67,
73, 75, 85, 89, 101, 103, 105, 109, 111, 113, 125, 127, 129

<u>Ex</u> Evaluate

1) $\dfrac{2.5 \times 10^{-3}}{5 \times 10^{2}} = \dfrac{25 \times 10^{-4}}{5 \times 10^{2}} = \dfrac{(25)(10^{-4})}{(5)(10^{2})} = 5 \cdot 10^{-4-2}$

$= 5 \cdot 10^{-6}$

$= 5 \times 10^{-6}$

2) $(2 \times 10^{9})(3.4 \times 10^{-4}) = (2)(10^{9})(3.4)(10^{-4})$

$= (6.8)(10^{9-4})$

$= 6.8 \times 10^{5}$