4.2 - Linear systems in two variables

Consider the system of equation

$$\begin{aligned} (1) \quad & 2x + y = 4 \\ (2) \quad & x - y = 2 \end{aligned}$$

→ we learned in the last section how to solve this using substitution. Today we'll learn a new method.

→ Remember that when we're solving a linear equation we can add the same thing to both sides of the equation and we have an equivalent equation.

→ Equation (2) above says $x-y=2$. So if we add $x-y$ on the left side of equation (1) and 2 on the right side, we have added the same thing to both sides of equation (1) and get an equivalent equation as the result.

(1) + (2) is

$$\begin{array}{rrcl} (1) & 2x + y & = & 4 \\ (2) + & x - y & = & 2 \\ \hline (1)+(2) = (3) & 3x & = & 6 \end{array}$$

→ By adding the two equations together we have eliminated y and gotten an equivalent equation that we can solve for x.

$$3x = 6 \Rightarrow \boxed{x=2}$$

→ We can substitute this back into equation (1) or (2) to solve for y.

$$2 - y = 2 \Rightarrow -y = 0 = \boxed{y=0}$$

So (2,0) is the solution to the system.

$\rightarrow$ This is called the method of elimination.

Suppose we have

$$\begin{array}{ll} (1) & 5x + 2y = 7 \\ (2) & -3x + y = -13 \end{array}$$

we know, we can multiply both sides of an equation by the same number and we'll get an equivalent equation. Multiply equation (2) by 2 to get equation (3)

$$(3) \quad -6x + 2y = -26$$

Now if we do equation (1) - equation (3), we have

$$\begin{array}{ll} (1) & 5x + 2y = 7 \\ (3) & -6x + 2y = -26 \\ (1)-(3)=(4) & 11x = 33 \end{array}$$

so $\boxed{x = 3}$

Then $-3(3) + y = -13$

$\rightarrow -9 + y = -13$

$\rightarrow \boxed{y = -4}$

→ Sometimes we have to do this process on both equations.

Ex Solve

(1) $2x + 3y = 8$

(2) $3x + 4y = 13$

(3) $6x + 9y = 24$ → $3 \cdot (1)$

(4) $6x + 8y = 26$ → $2 \cdot (2)$

(5) $\boxed{y = -2}$ → $(3) - (4)$

$2x + 3(-2) = 8$

$\Rightarrow 2x - 6 = 8$

$\Rightarrow 2x = 14$

$\Rightarrow \boxed{x = 7}$

Ex (1) $12x - 5y = 2$

(2) $-24x + 10y = 6$

(3) $24x - 10y = 4$ → $2 \cdot (1)$

(4) $0 = 10$ → $(2) + (3)$

No solution

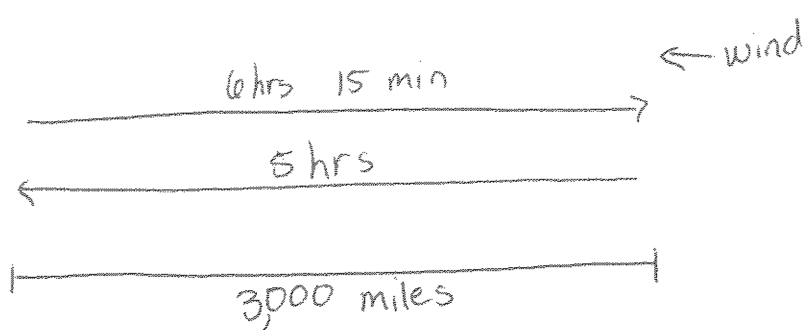
Ex
(1) $x - 3y = 9$
(2) $-3x + 9y = -27$

(3) $3x + 9y = 27$ → $3 \cdot (1)$

(4) $0 = 0$ → $(2) + (3)$

Infinitely many solutions

Ex Airplane flying into headwind travels 3,000 miles in 6 hours and 15 minutes. On the return trip, the plane has a tailwind and does the 3,000 miles in 5 hours. Find the speed of the plane in still air and the speed of the wind.



We know rate · time = distance
Let s = plane speed, w = wind speed

(1) $(s - w)\,6.25 = 3{,}000$

(2) $(s + w)\,5 = 3{,}000$

(3) $6.25s - 6.25w = 3{,}000$ → distribute in (1)

(4) $5s + 5w = 3{,}000$ → distribute in (2)

(5) $31.25s - 31.25w = 15000$ $\rightarrow 5 \cdot (3)$

(6) $31.25s + 31.25w = 18750$ $\rightarrow 6.25 \cdot (4)$

(7) $62.50s = 33750$ $\rightarrow (5)+(6)$

$\Rightarrow \boxed{s = 540}$

$5(540) + 5w = 3{,}000$

$\Rightarrow 2700 + 5w = 3{,}000$

$\rightarrow 5w = 300$

$\rightarrow \boxed{w = 60}$

plane
Air speed = 540 miles per hour

wind speed = 60 miles per hour

Ex You are hiking in the woods when an angry grizzly bear begins to chase you. You run for 2 hours at 10 miles per hour. How much longer must you run at 15 miles per hour order for the average speed for the whole run to be 12 miles per hour.

$\rightarrow$ remember $\dfrac{\text{total distance}}{\text{total time}} = \text{average speed}$

total distance = d
total time = time at 10mph + time at 15mph = $2+t$
Average rate = 12

(1) $12(t+2) = d$

And distance at 10mph + distance at 15mph = total distance

So

(2) $10(2) + 15(t) = d$

Then

(3) $12t + 24 = d$ → distribute (1)

(4) $15t + 20 = d$ → distribute (2)

(5) $3t - 4 = 0$ → (4) − (3)

$\Rightarrow 3t = 4$

$\Rightarrow t = \frac{4}{3}$ hours

= 1 hour and 20 minutes at 15 mph

Additional Practice Problems from Book, PP. 236-240

17, 19, 21, 23, 27, 31, 35, 41, 43, 45, 49, 51, 53, 61, 63, 67,
69, 71, 75, 77