4.1 - Systems of Equations

→ We have spent lots of time solving linear equations with one variable.

→ When we have more than 1 variable, we generally need more than one equation.

→ We'll focus on systems of linear equations.

→ Consider the system of equations

$$x + 2y = 9$$
$$-2x + 3y = 10$$

→ A solution is an ordered pair that makes both equations true simultaneously.

→ is $(3,3)$ a solution?

$3 + 2(3) = 9$ but $-2(3) + 3(3) = 3$, not 10

so $(3,3)$ is not a solution

→ $(1,4)$ is a solution

$$1 + 2(4) = 9$$
$$-2(1) + 3(4) = 10$$

→ We want a better way to solve besides guessing.

↳ method of substitution is one way

1) Solve for one variable in one equation

$$x + 2y = 9 \Rightarrow x = 9 - 2y$$

2) Substitute into the other equation and solve

$-2(9-2y)+3y=10$

$\Rightarrow -18+4y+3y=10$

$\Rightarrow 7y=28$

$\Rightarrow \boxed{y=4}$

3) solve for the other variable by substituting y-value

$x=9-2(4)$

$\Rightarrow x=9-8$

$\Rightarrow \boxed{x=1}$

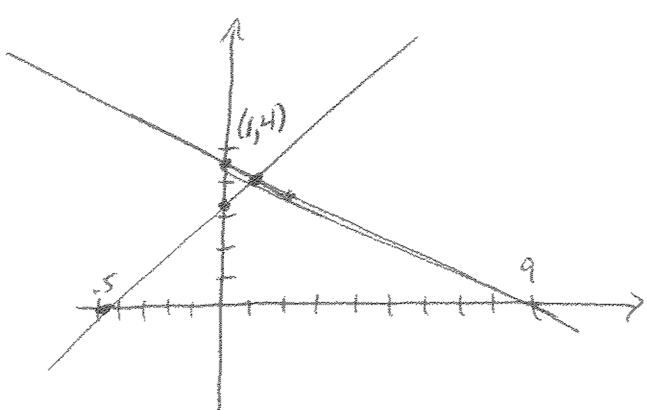
(1,3) is the solution

4) Check the answer!

we could also solve the system graphically

$x+2y=9 \Rightarrow y=-\frac{1}{2}x+\frac{9}{2}$

$-2x+3y=10 \Rightarrow y=\frac{2}{3}x+\frac{10}{3}$



→ The solution to the system is ~~the~~ the intersection of the two lines.

→ This graphical method is generally not the best/easiest way to solve these types of equations.

Ex Solve the system of equations

$x - 2y = 0$
$3x + 2y = 8$

$x = 2y$ → solve

$3(2y) + 2y = 8$ → substitute & solve

$6y + 2y = 8$

$8y = 8$

$\boxed{y = 1}$

$x = 2(1)$ → substitute

$\boxed{x = 2}$

$(2,1)$ → solution

Ex Solve

$3x + y = 8$
$3x + y = 6$

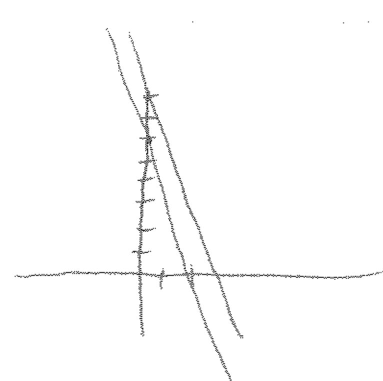
$y = 8 - 3x$ → solve

$3x + 8 - 3x = 6$ → substitute & solve

$8 \neq 6$ → no solution

This system is inconsistent

Graphically: $y = -3x + 8$
$y = -3x + 6$



The lines are parallel!

Ex Solve

$2x - \cancel{8}4y = 6$

$x - 2y = 3$

$4y = 2x - 6 \Rightarrow y = \frac{1}{2}x - \frac{3}{2}$

$2y = x - 3 \Rightarrow y = \frac{1}{2}x - \frac{3}{2}$

→ It's really the same line!

$x = 2y + 3$

$2(2y+3) - 4y = 6$

$4y + 6 - 4y = 6$

$6 = 6$

Infinitely many solutions

3 cases

(1) 1 solution → 2 lines with different slopes

(2) no solutions → 2 parallel lines

(3) Infinitely many solutions → Only 1 line

Ex Business invests \$ ~~50~~ 10,000 in equipment to make pink fuzzy valentine bears. The bears sell for \$3.25 each and cost \$1.65 to make. How many bears must be sold for the business to break even?

→ want cost to be equal to revenue.

$C = 10{,}000 + 1.65x$

$R = 3.25x$

$10{,}000 + 1.65x = 3.25x$

$10{,}000 = 1.60x$

$6250 = x$

<u>Ex</u> You buy your Valentine a total of 80 chocolates and red roses. You buy her 18 more roses than chocolates. How many do you buy of each?

$c + r = 80$

$r - c = 18$

$r = c + 18$

$c + c + 18 = 80$

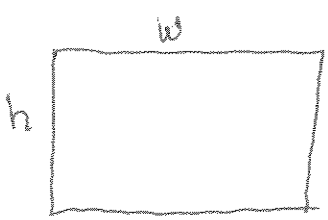
$2c = 62$

$\boxed{c = 31}$

$r = 31 + 18$

$\boxed{r = 49}$

<u>Ex</u> A rich wife buys her sports-loving husband a flat screen tv for Valentine's Day. The perimeter of the tv is 280 inches. Three times the height of the tv is equal to twice the width. What are the dimensions of the tv?



$2w + 2h = 280$

$3h = 2w$

$h = \frac{2w}{3}$

$2w + 2\left(\frac{2w}{3}\right) = 280$

$2w + \frac{4w}{3} = 280$

$\frac{6w}{3} + \frac{4w}{3} = 280$

$\frac{10w}{3} = 280$

$10w = 840$

$\boxed{w = 84}$

$h = \frac{2(84)}{3} = \boxed{56}$

Section 4.1-Systems of Equations Supplemental Problems. Pages 218-230
Problems numbers: 9, 11, 13, 27, 29, 31, 35, 37, 53, 55, 61, 67, 87, 89, 95, 97, 105 107, 111, 113, 117, 119.