

3.6 - Relations and Functions

①

→ Consider a ^(specific) parking meter. For a specific input (quarter, nickel, dime, etc.) you get a specific output (20 minutes, 2 minutes, 8 minutes, etc.). The parking meter is useful because for a specific input, we ~~know~~ know what we'll get out. If, for example, you put in a quarter and sometimes you got 20 minutes and other times you got 3 minutes, the parking meter would be less useful.

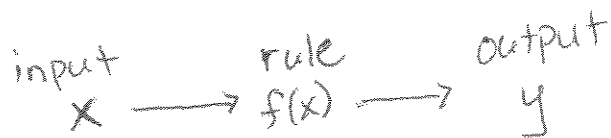
→ Now consider the equation $y = x^2$. For a specific x -value input you know exactly what y -value output you will get.

▷ we say that y is a function of x

↳ we write $y = f(x) = x^2$

↳ f is the name of the function

→ here's how functions work:



3 ingredients: i) Domain

set of all possible inputs

ii) Range

set of all possible outputs

iii) Rule

tells us what to do with the input to compute the output

EX

Rule

Domain

Range

②

$$f(x) = x^2$$

$$(-\infty, \infty)$$

$$[0, \infty)$$

$$f(x) = \frac{1}{x}$$

$$(-\infty, 0), (0, \infty)$$

$$(-\infty, 0), (0, \infty)$$

$$f(x) = \sqrt{x}$$

$$[0, \infty)$$

$$[0, \infty)$$

EX Find the domain of

$$1) \frac{2}{(x-2)(x+3)}$$

→ we don't want to divide by zero

$$x-2=0 \Rightarrow x=2$$

so we don't want this

$$x+3=0 \Rightarrow x=-3$$

so we don't want this.

→ All other numbers are fine, so the domain is all real numbers x such that $x \neq 2, x \neq -3$

$$2) \sqrt{2x+1}$$

→ can't take the square root of a negative number.

$$\rightarrow \text{want } 2x+1 \geq 0$$

$$\Rightarrow 2x \geq -1$$

$$\Rightarrow x \geq \frac{1}{2}$$

$$\text{or } x \in \left[\frac{1}{2}, \infty\right)$$

→ Suppose $f(x) = 2x + 5$. $f(2)$ means apply rule to find output when input is 2.

(3)

→ Write $f(2) = 2(2) + 5 = 4 + 5 = 9$

EX Let $f(x) = \frac{x}{x^2 + 1}$

Domain: $(-\infty, \infty)$ range: $(-\infty, \infty)$

a) Evaluate $f(3)$

$$f(3) = \frac{3}{3^2 + 1} = \frac{3}{9 + 1} = \frac{3}{10}$$

b) Evaluate $f(-2)$

$$f(-2) = \frac{-2}{(-2)^2 + 1} = \frac{-2}{4 + 1} = \frac{-2}{5}$$

c) Evaluate $f(t+1)$

$$\begin{aligned} f(t+1) &= \frac{t+1}{(t+1)^2 + 1} \\ &= \frac{t+1}{(t^2 + 2t + 1) + 1} \\ &= \frac{t+1}{t^2 + 2t + 2} \end{aligned}$$

Aside: $(t+1)^2 = (t+1)(t+1)$

FOIL: $t^2 + t + t + 1$
 $= t^2 + 2t + 1$

EX Let $f(x) = 2x + 5$

a) Evaluate $f(2) + f(3)$: $f(2) + f(3) = 2(2) + 5 + 2(3) + 5 = 4 + 5 + 6 + 5 = 20$

b) Evaluate $f(f(x))$: $f(f(x)) = 2(2x + 5) + 5$

$$\begin{aligned} &= 4x + 10 + 5 \\ &= 4x + 15 \end{aligned}$$