3.6 - Relations and Functions

→ Consider a (specific) parking meter. For a specific input (quarter, nickel, dime, etc.) you get a specific output (20 minutes, 2 minutes, 8 minutes, etc.). The parking meter is useful because for a specific input, we know what we'll get out. If, for example, you put in a quarter and sometimes you got 20 minutes and other times you got 3 minutes, the parking meter would be less useful.

→ Now consider the equation $y = x^2$. For a specific x-value input you know exactly what y-value output you will get.

▷ we say that y is a function of x

↳ we write $y = f(x) = x^2$

↳ f is the name of the function

→ here's how functions work:

$$\underset{x}{\text{input}} \longrightarrow \underset{f(x)}{\text{rule}} \longrightarrow \underset{y}{\text{output}}$$

3 ingredients:

- i) Domain — set of all possible inputs
- ii) Range — set of all possible outputs
- iii) Rule — tells us what to do with the input to compute the output

Ex

Rule	Domain	Range
$f(x) = x^2$	$(-\infty, \infty)$	$[0, \infty)$
$f(x) = \frac{1}{x}$	$(-\infty, 0), (0, \infty)$	$(-\infty, 0), (0, \infty)$
$f(x) = \sqrt{x}$	$[0, \infty)$	$[0, \infty)$

Ex Find the domain of

1) $\frac{2}{(x-2)(x+3)}$ → we don't want to divide by zero

$x-2=0 \Rightarrow x=2$ so we don't want this

$x+3=0 \Rightarrow x=-3$ so we don't want this.

→ All other numbers are fine, so the domain is all real numbers x such that $x \neq 2$, $x \neq -3$

2) $\sqrt{2x+1}$ → can't take the square root of a negative number.

→ want $2x+1 \geqslant 0$

$\Rightarrow 2x \geqslant -1$

$\Rightarrow x \geqslant \frac{1}{2}$

or $x \in [\frac{1}{2}, \infty)$

→ Suppose $f(x) = 2x+5$. $f(2)$ means apply rule to find output when input is 2.

→ Write $f(2) = 2(2)+5 = 4+5 = 9$

Ex Let $f(x) = \dfrac{x}{x^2+1}$

Domain: $(-\infty, \infty)$ range: $(-\infty, \infty)$

a) Evaluate $f(3)$

$$f(3) = \frac{3}{3^2+1} = \frac{3}{9+1} = \frac{3}{10}$$

b) Evaluate $f(-2)$

$$f(-2) = \frac{-2}{(-2)^2+1} = \frac{-2}{4+1} = -\frac{2}{5}$$

c) Evaluate $f(t+1)$

$$f(t+1) = \frac{t+1}{(t+1)^2+1}$$

$$= \frac{t+1}{(t^2+2t+1)+1}$$

$$= \frac{t+1}{t^2+2t+2}$$

Aside: $(t+1)^2 = (t+1)(t+1)$

FOIL: $t^2+t+t+1$

$= t^2+2t+1$

Ex Let $f(x) = 2x+5$

a) Evaluate $f(2)+f(3)$: $f(2)+f(3) = 2(2)+5 + 2(3)+5 = 4+5+6+5 = 20$

b) Evaluate $f(f(x))$: $f(f(x)) = 2(2x+5)+5$

$= 4x+10+5$

$= 4x+15$