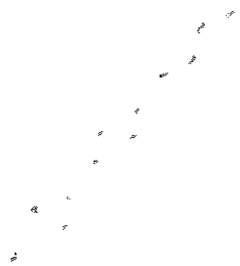
3.4 - Equations of Lines

→ suppose we have some data



→ we can approximate this data with a straight line, but we won't get all the points.

→ If we only had 2 data points, we could draw a line through those data points

Ex find equation of line through $\underset{(x_1,\,y_1)}{(-2,1)}$ and $\underset{(x_2,\,y_2)}{(2,-3)}$

↳ we need the slope first

$$m = \frac{\text{rise}}{\text{run}} = \frac{-3-1}{2-(-2)} = \frac{-4}{4} = -1$$

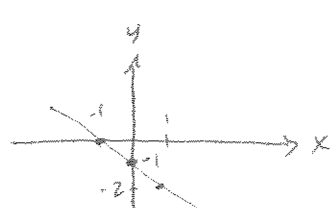
↳ If we calculate using any other point (x,y) on the line, we should get the same slope.

↳ Use general point (x,y) and one of the points we know

$$\frac{y-1}{x-(-2)} = -1 \Rightarrow \frac{y-1}{x+2} = -1 \Rightarrow y-1 = -x-2$$

$$\Rightarrow y = -x-1$$

→ slope intercept form

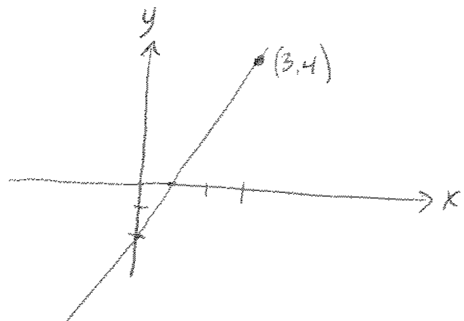


→ Suppose we know the slope of a line, $m=2$ and one point on the line, $(3,4)$. Find the equation.

→ This is like the 2nd part of the previous example.

→ Calculating the slope between any general point on the line and (x,y) and the point we know $(3,4)$ should yield the slope of 2.

$$\frac{y-4}{x-3}=2 \Rightarrow y-4=2(x-3) \Rightarrow y-4=2x-6 \Rightarrow y=2x-2$$



point slope form:

$$y-y_1=m(x-x_1)$$

→ What's the equation of a horizontal line passing through the point $(3,6)$?

↳ horizontal means same y value regardless of the x value.

Answer: $y=6$ (slope of 0)

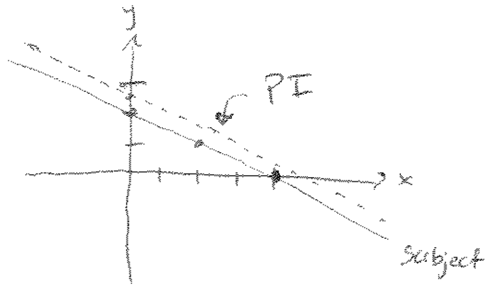
→ A vertical line passing through $(3,6)$?

↳ vertical means same x value regardless of y value.

Answer: $x=3$ (slope undefined)

Ex A private detective (or creepy stalker) walks in a straight line parallel to their subject who walks in a straight line given by $2x+4y=8$. The PI passes through the point $(3,1)$. What's the equation of his line?

→ Subject's line $2x+4y=8 \Rightarrow 4y=-2x+8 \Rightarrow y=-\frac{1}{2}x+2$



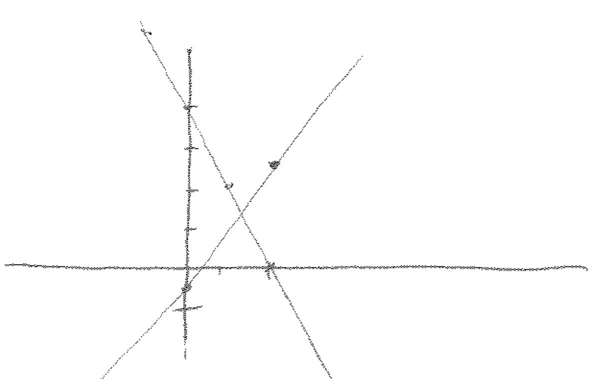
→ slope of PI's line? parallel to subject, so $m=-\frac{1}{2}$

↳ Equation $\dfrac{y-1}{x-3}=-\frac{1}{2} \Rightarrow y-1=-\frac{1}{2}x+\frac{3}{2} \Rightarrow y=-\frac{1}{2}x+\frac{5}{2}$

Ex An out of control longboarder skates in straight line given by $3x-2y=1$. An unsuspecting walker walks in a straight line given by $2x+y=4$. What are their coordinates when they collide?

longboarder: $3x-2y=1 \Rightarrow -2y=-3x+1 \Rightarrow y=\frac{3}{2}x-\frac{1}{2}$

walker: $2x+y=4 \Rightarrow y=-2x+4$



→ when they collide, their x and y values must be the same.

→ $\frac{3}{2}x-\frac{1}{2}=-2x+4$

$\frac{3}{2}x+2x=4+\frac{1}{2}$

$\frac{3}{2}x+\frac{4}{2}x=\frac{8}{2}+\frac{1}{2}$

$\frac{7}{2}x=\frac{9}{2}$

→ $7x=9$

$x=\frac{9}{7}$

$y=-2\left(\frac{9}{7}\right)+4$

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→ when they collide, their x and y values are the same

↳ set RHS of equations equal to each other & solve for x

$$\frac{3}{2}x - \frac{1}{2} = -2x + 4$$

$$\frac{3}{2}x + 2x = 4 + \frac{1}{2}$$

$$\frac{3}{2}x + \frac{4}{2}x = \frac{8}{2} + \frac{1}{2}$$

$$\frac{7}{2}x = \frac{9}{2}$$

$$7x = 9$$

$$x = \frac{9}{7}$$

→ So then we can find y using either equation.

$$y = -2\left(\frac{9}{7}\right) + 4$$

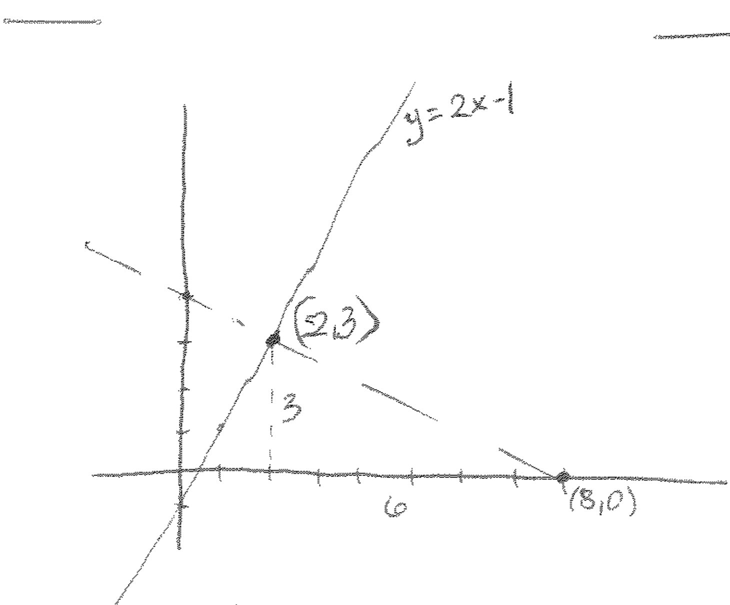
$$= \frac{-18}{7} + 4$$

$$= \frac{-18}{7} + \frac{28}{7}$$

$$y = \frac{10}{7}$$

→ they collide at $\left(\frac{9}{7}, \frac{10}{7}\right)$

<u>Ex</u> There is a busy street where traffic travels along the line $y = 2x - 1$. A toddler stumbles towards the street. Spiderman swings in, shoots a sticky ~~[illegible]~~ web at the child, and immobalizes him on the point $(8,0)$. How close is the child to the traffic?



→ we want to find the perpendicular distance.

→ find line perpendicular to $y=2x-1$ through point $(8,0)$

→ slope $= -\frac{1}{2}$

→ $\frac{y-0}{x-8} = -\frac{1}{2}$ ⇒ $\boxed{y = -\frac{1}{2}x + 4}$

↳ we need to find the point of intersection

$2x - 1 = -\frac{1}{2}x + 4$

$2x + \frac{1}{2}x = 4 + 1$

$\frac{4}{2}x + \frac{1}{2}x = 5$

$\frac{5}{2}x = 5$

$5x = 10$

$x = 2$

→ so then $y = -\frac{1}{2}(2) + 4$

$= -1 + 4$

$= 3$

pt of intersection $(2,3)$

Finally the distance, → use right triangles

$d^2 = 3^2 + 6^2 = 9 + 36 = 45$

so $d = \sqrt{45} = 3\sqrt{5} \approx 6.708$