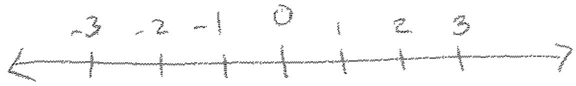
2.4| Linear inequalities

Recall the real number line



→ we order numbers on the real number line with inequalities.

$a > b$ a is to the right of b
greater than

$a \geq b$ a is greater than or equal to b

$a < b$ a is to the left of b
less than

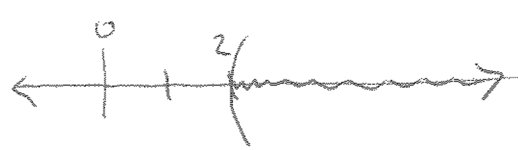
$a \leq b$ a is less than or equal to b

→ Suppose $x+1 > 3$. what do we know about x? $x > 2$

element of

Another way to write this is $x \in (2, \infty)$ → parentheses for strict inequalities

Graphically

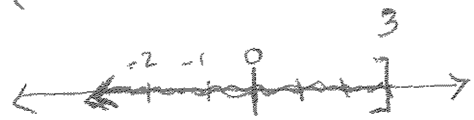


→ These ranges of possible x values are called <u>intervals</u>

→ Suppose $x - 1 \le 2$. What do we know about x?

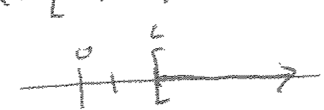
$x \le 3$ → brackets for nonstrict inequalities

or $x \in (-\infty, 3]$

or 

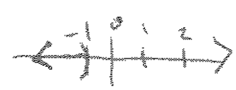
→ Suppose $2x \ge 4$. Then $x \ge 2$

or $x \in [2, \infty)$

or 

→ Suppose $-x > 1$. Then $x < -1$. Tricky!

or $x \in (-\infty, -1)$

or 

☆ To solve inequalities we proceed the same way as with equations except we must reverse the direction of the inequality if we multiply or divide by a negative number.

Notice: $1 < 5$. Now multiply both sides by -1

$-1 > -5$

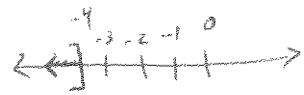
Ex

$$8 - 3x \le 20 \qquad \rightarrow +3x - 20$$

$$-12 \le 3x \qquad \rightarrow \div 3$$

$$-4 \le x$$

or $x \in (-\infty, -4]$ or



Alternatively

$$8 - 3x \le 20 \qquad \rightarrow -8$$

$$-3x \le 12 \qquad \rightarrow \div -3 \quad \text{(switch inequality)}$$

$$x \ge -4$$

Ex

$$7x - 3 > 3(x+1) \qquad \rightarrow \text{distribute}$$

$$7x - 3 > 3x + 3 \qquad \rightarrow +3 - 3x$$

$$4x > 6 \qquad \rightarrow \div 4$$

$$x > \frac{6}{4} \qquad \rightarrow \text{simplify}$$

$$x > \frac{3}{2}$$

or $x \in \left(\frac{3}{2}, \infty\right)$

Ex

$$\frac{2x}{3} + 12 < \frac{x}{6} + 18 \qquad \rightarrow -12 - \frac{x}{6}$$

$$\frac{2x}{3} - \frac{x}{6} < 6 \qquad \rightarrow \text{CD}$$

$$\frac{4x}{6} - \frac{x}{6} < 6 \qquad \rightarrow \text{combine like terms}$$

$$\frac{3x}{6} < 6 \qquad \rightarrow \cdot 6$$

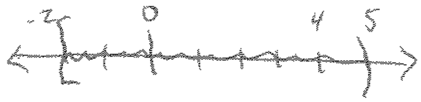
$$3x < 36 \qquad \rightarrow \div 3$$

$$x < 12$$

Suppose $x < 5$ and $x \geq -2$. We can write

$-2 \leq x < 5$ both must be true

or $[-2, 5)$

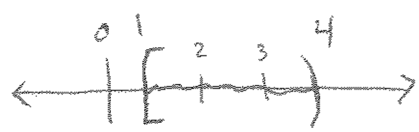
or 

We can also solve combined inequalities

Ex $-1 \leq 2x - 3 < 5$ $\rightarrow +3$

$2 \leq 2x < 8$ $\rightarrow \div 2$

$1 \leq x < 4$

or $x \in [1, 4)$ or 

Ex $-3 < 5 - 2x \leq 1$ $\rightarrow -5$

$-8 < -2x \leq -4$ $\rightarrow \div(-2)$

$4 > x \geq 2$

or $2 \leq x < 4$

$x \in [2, 4)$

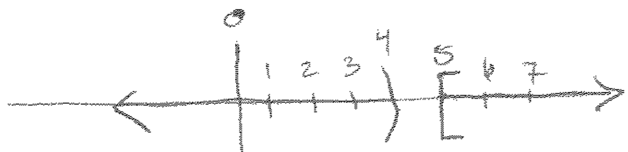
Suppose $2x - 3 < 5$ or $2x - 3 \geq 7$

↳ only one is true

$$2x - 3 < 5 \quad \text{or} \quad 2x - 3 \geq 7$$

$$2x < 8 \qquad\qquad 2x \geq 10$$

$$x < 4 \qquad\qquad x \geq 5$$



↳ in this case we solve the inequalities separately