

1. You spend an evening at the circus and are delighted to see that Bubba, also known as “THE AMAZING FLYING SQUIRREL,” is going to be shot out of a cannon into a small pool of water embedded in the floor. The pool is 100 feet from the cannon. Being a student of science, you pull out a pencil and paper and quickly begin to calculate to see if Bubba will in fact land in the pool of water or if the night will end in tragedy. (You happen to live in a special world that has no air resistance.) The end of Bubba’s cannon is 4 feet above the ground. You hear the ringmaster say that upon leaving the cannon, the horizontal component of Bubba’s initial velocity is 25 feet per second and that the vertical component of Bubba’s velocity is  $25\sqrt{3}$  feet per second. You recall a fact from trigonometry and remember an old Greek mathematician (Pythagoras), and realize that Bubba will be shot from the cannon at an angle of  $60^\circ$  with a velocity of 50 feet per second. You understand that this information isn’t needed to solve your problem, but being a good student with an enquiring mind, you revel in any increase in knowledge. Then you get down to business. Will Bubba land in the pool?

**Solution:** You recall from physics that the equation

$$h(t) = -16t^2 + v_0t + h_0$$

describes an object’s height at time  $t$ . In Bubba’s case, the equation is

$$h(t) = -16t^2 + 25\sqrt{3}t + 4$$

Since the pool is at a height of zero feet, setting  $h(t) = 0$  and solving should tell you the time Bubba spends flying through the air. Using this time, along with the given horizontal rate, you can calculate the distance flown via the equation: distance = (rate)(time). So then

$$\begin{aligned} 0 = -16t^2 + 25\sqrt{3}t + 4 &\Rightarrow 0 = t^2 - \frac{25\sqrt{3}}{16}t - \frac{4}{16} \\ &\Rightarrow t^2 - \frac{25\sqrt{3}}{16}t - \frac{1}{4} = 0 \\ &\Rightarrow t^2 - \frac{25\sqrt{3}}{16} = \frac{1}{4} \\ &\Rightarrow t^2 - \frac{25\sqrt{3}}{16} + \left(\frac{25\sqrt{3}}{32}\right)^2 = \frac{1}{4} + \left(\frac{25\sqrt{3}}{32}\right)^2 \\ &\Rightarrow \left(t - \frac{25\sqrt{3}}{32}\right)^2 = \frac{1}{4} + \frac{1875}{1024} \\ &\Rightarrow \left(t - \frac{25\sqrt{3}}{32}\right)^2 = \frac{2131}{1024} \end{aligned}$$

$$\begin{aligned} \Rightarrow t - \frac{25\sqrt{3}}{32} &= \pm \sqrt{\frac{2131}{1024}} \\ \Rightarrow t - \frac{25\sqrt{3}}{32} &= \pm \frac{\sqrt{2131}}{32} \\ \Rightarrow t &= \frac{25\sqrt{3}}{32} \pm \frac{\sqrt{2131}}{32} \end{aligned}$$

So you have found two times corresponding to when Bubba hits the ground (hopefully in the pool). You realize that negative time doesn't make sense, so you take the positive time, which is

$$t = \frac{25\sqrt{3} + \sqrt{2131}}{32} \approx 2.796 \text{ seconds}$$

You recognize that Bubba's horizontal velocity will remain constant while he is flying through the air and calculate that he will fly horizontally

$$(25) \left( \frac{25\sqrt{3} + \sqrt{2131}}{32} \right) \approx 69.9 \text{ feet}$$

You gasp as you finish the calculation, realizing that Bubba is headed towards an untimely death. By this time though, the ringmaster is in the process of lighting the cannon's igniter. The cannon roars and you close your eyes, not wishing to observe the grisly scene. You hear an ooh of excitement, though, and open your eyes to see that Zelda, the talented female trapeze artist has caught Bubba midflight. Zelda tosses Bubba to Mo, another trapeze artist, who drops Bubba, KERPLUNK!, right into the pool of water. The crowd goes wild, completely unaware of Bubba's close brush with death, and you spend the rest of the evening feasting on caramel corn and roasted peanuts. You go home and dream of one day being a math teacher and writing interesting story problems.

2. You and your young brother Timmy spend a lot of time playing in your back yard. It's a pretty sweet place except for the well. You've lost many a toy over the years to that well since it has no walls or fences around it. The well is too dark inside to see the bottom, and you've always wondered just how deep it is. On this fine afternoon, you and Timmy are playing a game you affectionately call, "Who can walk closest to the well without falling in and drowning?" The game gets heated and Timmy walks just a little bit too far and starts falling into the well. This is unfortunate for Timmy, but great for you because you realize you have the perfect opportunity to answer the question of just how deep the well is. You whip out your stop watch and press start just as Timmy tips into the gaping hole. You click your stop watch when you hear a splash and it reads  $\frac{3}{2}$  seconds. Now you set about calculating the depth of the well.

**Solution:** You pretend that you live in a special world where sounds move at infinite speed. That

means that you hear a splash the instant that Timmy hits the water. You remember from physics that Timmy's height above the water at the bottom of the well is given by

$$h(t) = -16t^2 + v_0t + h_0$$

Now, you didn't throw Timmy down into the well, though you've wanted to on many occasions, so his initial vertical velocity is zero. You don't know his initial height above the water (this is what you're trying to figure out), but you do know that he hits the water after  $\frac{3}{2}$  seconds. This means that  $h(t) = 0$  when  $t = \frac{3}{2}$ . This gives you the equation

$$\begin{aligned} 0 &= -16 \left(\frac{3}{2}\right)^2 + h_0 \Rightarrow 0 = -16 \left(\frac{9}{4}\right) + h_0 \\ &\Rightarrow 0 = -36 + h_0 \\ &\Rightarrow h_0 = 36 \end{aligned}$$

So in your pretend world, the well is 36 feet deep.

You realize that in the real world though, sound moves at a finite speed of around 1100 feet per second. This means that since the sound had to travel  $h_0$  feet to get to you and distance=(rate)(time), it took the sound  $h_0 = 1100t_t \Rightarrow t_t = 1100/h_0$  seconds to reach you. During this time, Timmy was already in the water. So if  $t$  is the number of second that Timmy actually spent falling through the air and  $t_t$  is the additional amount of time it took the splash to reach you, the sum of those two times should be the  $\frac{3}{2}$  second that you have on your stopwatch. Thus, Timmy actually fell for  $\frac{3}{2} - t_t$  seconds. You repeat your calculation with this added information:

$$\begin{aligned} 0 &= -16 \left(\frac{3}{2} - t_t\right)^2 + h_0 \Rightarrow 0 = -16 \left(\frac{3}{2} - t_t\right)^2 + 1100t_t \\ &\Rightarrow 0 = -16 \left(\frac{9}{4} - 3t_t + t_t^2\right) + 1100t_t \\ &\Rightarrow 0 = -36 + 48t_t - 16t_t^2 + 1100t_t \\ &\Rightarrow -16t_t^2 + 1148t_t - 36 = 0 \end{aligned}$$

Using the quadratic formula, you find that

$$\begin{aligned} t_t &= \frac{-1148 \pm \sqrt{1148^2 - 4(-16)(-36)}}{2(-16)} \\ &= \frac{-1148 \pm \sqrt{1317904 - 2304}}{-32} \end{aligned}$$

$$\begin{aligned}
&= \frac{-1148 \pm \sqrt{1315600}}{-32} \\
&= \frac{-1148 \pm 20\sqrt{3289}}{-32} \\
&= \frac{287}{8} \pm \frac{5\sqrt{3289}}{8}
\end{aligned}$$

You realize that these are two positive times, but one of them is very large, making  $\frac{3}{2} - t_t$  negative. This tells you that

$$t_t = \frac{287}{8} - \frac{5\sqrt{3289}}{8} \approx 0.031373 \text{ seconds}$$

This means that the distance Timmy falls is

$$h_0 = (1100) \left( \frac{287}{8} - \frac{5\sqrt{3289}}{8} \right) \approx 34.5 \text{ feet.}$$

You are elated to finally know the depth of that pesky well. Slightly less exciting is the fact that you have a 40 foot long rope. You throw the rope to Timmy and haul him out. You become an immediate media sensation both for your mathematical abilities and for your good hearted nature in saving your annoying little brother from certain death. You go on the Today Show for an in-depth discussion of the quadratic formula and its applications. Lucky for you, and unlike “the man with the golden voice”, you don’t have a drug and alcohol problem and don’t disappear from the media spotlight after enrolling in rehab.