Pseudo-Exam - Calculus III, Math 2210, Sect. 5
December 11, 2008

Exercise 1. Let $f(x,y) = 2e^{-x^2-y^2} - 3x^2 - 3y^2$.

(1) Find all **local** minima and maxima of f on its domain.
(2) Find all **global** minima and maxima of f on its domain.
(3) Find the tangent plane to the surface $z = f(x,y)$ at $(x_0, y_0) = (1,0)$ and find its normal vector (i.e. the vector orthogonal to the tangent plane and of magnitude 1).
(4) Evaluate

$$\int_S f(x,y)\, dxdy,$$

where

$$S = \left\{(x,y)\ :\ x^2+y^2 \leq 9,\ y \geq 0\right\}.$$

(5) Evaluate

$$\int_C \nabla f \cdot d\mathbf{r},$$

where

$$C\ :\ \begin{cases} x = \log(t^2+1) \\ y = \frac{t^6}{(e-1)^3} \end{cases} \qquad 0 \leq t \leq \sqrt{e-1}.$$

Exercise 2. Let $\mathbf{F}(x,y,z) = \cos(xy)\,\mathbf{i} + \sin(xy)\,\mathbf{j} + z\,\mathbf{k}$.

(1) Compute $\nabla \cdot \mathbf{F}$ and $\nabla \times \mathbf{F}$.
(2) Is $\mathbf{F}$ conservative?
(3) Evaluate

$$\int_C \mathbf{F} \cdot d\mathbf{r},$$

where

$$C\ :\ \begin{cases} x = t \\ y = 1 \\ z = e^t \end{cases} \qquad 0 \leq t \leq 1.$$

Exercise 3. Let

$$C\ :\ \begin{cases} x = \cos(t) \\ y = \sin(t) \\ z = \cos^2(t) \end{cases} \qquad 0 \leq t \leq 2\pi$$

be a curve and let $\mathbf{F}(x, y, z) = -y\,\mathbf{i} + x\,\mathbf{j} - 2xy\,\mathbf{k}$ be a vector field.

(1) Show that C is a smooth closed curve in 3D.

(2) Show that

$$\langle x'(t), y'(t), z'(t)\rangle = \mathbf{F}(x(t), y(t), z(t)).$$

(3) Evaluate

$$\oint_C \mathbf{F} \cdot d\mathbf{r}.$$

(4) (facultative) Let

$$C\ :\ \begin{cases} x = x(t) \\ y = y(t) \\ z = z(t) \end{cases} \qquad a \leq t \leq b$$

be a **smooth, closed** curve in 3D and let $\mathbf{F}(x, y, z)$ be a vector field. Show that if

$$\langle x'(t), y'(t), z'(t)\rangle = \mathbf{F}(x(t), y(t), z(t))$$

then

$$\oint_C \mathbf{F} \cdot d\mathbf{r} > 0.$$