(1) Do again all previous homework exercises. In particular, those of Sections 2.1, 2.2, 2.3, 2.4, 3.3, 3.4, 3.5, 3.6.

(2) Find a solution $u = u(x, t)$ to the wave equation

$$u_{tt} = u_{xx} \quad (x = 1)$$

in the region $0 < x < 1$, $t > 0$

with boundary conditions

$$u(0, t) = 0 = u(1, t)$$

and initial conditions

$$u(x, 0) = e^x$$

$$u_t(x, 0) = \cos(x)$$
(3) Find a solution $u = u(x,t)$ to the wave equation

$$u_{tt} = u_{xx} \quad (x \in R)$$

in the region $0 < x < 1, \quad t > 0$

with boundary conditions

$$u(0,t) = 0$$
$$u(1,t) = 1 \quad (\text{note} = 1)$$

and initial conditions

$$u(x,0) = 2x$$
$$u_t(x,0) = 0$$

[Hint: Try to reduce to a problem you are able to solve in a similar way as we did for the heat equation]
(4) Find a solution $u(x,t)$ to the heat equation

$$u_t = 2u_{xx} \quad (x>0)$$

in the region $0 < x < 1$, $t > 0$.

with boundary conditions

$$u(0,t) = 0$$

$$u(1,t) = t$$ \quad (note: t)

and initial condition

$$u(x,0) = 0$$

[Hint: As in Exercise (3) try to reduce to a problem you are able to solve as we did in class]
(5) Find a solution \( u = u(x,t) \) to the heat equation

\[
  u_t = u_{xx} \quad (c=1)
\]

in the region \( 0 < x < 2, \ t > 0 \)

with boundary conditions

\[
  u(0,t) = 1 \\
  u(2,t) = 0
\]

and initial condition

\[
  u(x,0) = \frac{1}{2}
\]

Then evaluate \( u \) at \( x = 1 \) \( \text{“evaluate” means find a number!} \)

\[
  u(1,t) = ?
\]

[Hint: The first part of the exercise is standard: we have seen it in class.
For the second question, use symmetry: first solve

\[
  u_t = u_{xx} \\
  u(0,t) = u(2,t) = u(x,0) = 1
\]

and then argue by symmetry ...]

\[
  \Rightarrow \text{by looking at}
  \\
  u_t = u_{xx} \\
  u(0,t) = 0 \\
  u(2,t) = 1 \\
  u(x,0) = \frac{1}{2}
\]
(6) Use the method of separation of variables to find a solution to the problem $(u = u(x,t))$

\[ u + u_x - u_t = 0 \quad x, t \text{ real numbers} \]
\[ u(x,0) = 2e^{-2x} \]
\[ u(0,t) = 2e^{-t} \]

(7) Consider the PDE $(u = u(x,t))$

\[ [\ast] \quad u_t + uu_x + 6u u_x = 0 \]

Show that a solution can be found by first solving

\[ [\ast \ast] \quad -f' + f''' + 6f f' = 0 \quad (f = f(s)) \]

and then setting $u(x,t) = f(x-t)$.

Prove that $f(s) = \frac{1}{2} \frac{1}{\cosh^2(s/2)}$ is a solution of $[\ast \ast]$

and then find a solution of $[\ast]$.