Exercises

1) \[ f(x,y) = x^2 + 4xy + 5y^2 + 3 \]

- What is the domain of \( f(x,y) \)?
- What is the range of \( f(x,y) \)?
- Is \( f(x,y) \) continuous?
- Compute \( \nabla f(x,y) \)
- Find tangent plane to \( z = f(x,y) \) at \( (x_0, y_0) = (1,1) \)
- Explain why \( \frac{\partial}{\partial y} f(x,y) = \frac{\partial}{\partial x} f(x,y) \)
- Sketch the graph \( z = f(x,y) \)

2) Same as Ex.1, with \( f(x,y) = \sqrt{4-x^2-y^2} \)

3) Same as Ex.1, with \( f(x,y) = \log(x^2+y^2) \)
   - What happens at \( (x,y) = (0,0) \)?

4) Find an example of a function \( f(x,y) \) such that \( \frac{\partial}{\partial x} f(x,y) \) exists but \( \frac{\partial}{\partial y} f(x,y) \) does not at a certain point \( (x_0, y_0) \) in the domain of \( f(x,y) \).

5) Find an example of a function \( f(x,y) \) such that \( \frac{\partial}{\partial x} f(x,y) \) and \( \frac{\partial}{\partial y} f(x,y) \) exist at a point \( (x_0, y_0) \) in the domain of \( f(x,y) \) but the tangent plane does not make sense...
(6) \( \mathbf{r}(t) = \begin{bmatrix} z \cos(t/2) \\ \sqrt{2} \sin(t/2) \\ \sqrt{2} \sin(t/2) \end{bmatrix} \) curve in 3D, \( 0 \leq t \leq 4\pi \)

- Compute \( \| \mathbf{r}(t) \|, \mathbf{v}(t), \mathbf{v}(t), \mathbf{a}(t) \).
- Show that \( \mathbf{r}''(t) = -\frac{\mathbf{r}(t)}{\| \mathbf{r}(t) \|^2} \) for all \( 0 \leq t \leq 4\pi \).
- Use equation \( \ast \) to prove that 
  \[ \frac{d}{dt} \left[ \mathbf{r}'(t) \times \mathbf{r}(t) \right] = 0 \]
- Prove that all curves \( \mathbf{r}(t) \) which satisfy \( \ast \) are planar, i.e., they lie on a plane.

Is it true?

\( \ast \) Show that a curve \( \mathbf{r}(t) \) with curvature \( \kappa(t) \) constant is a circle if \( \kappa(t) \neq 0 \) and it is a line if \( \kappa(t) = 0 \) ?

Apply this to Ex. 6 (DIFFICULT).

From textbook: Go through all her done so far! Plus:

13.4: \( \text{all} \rightarrow 4, 11, 15, 16, 17, 28, 33 \)
13.5: \( \text{all} \rightarrow 4, 14, 18, 44, 45, 46, 49 \)