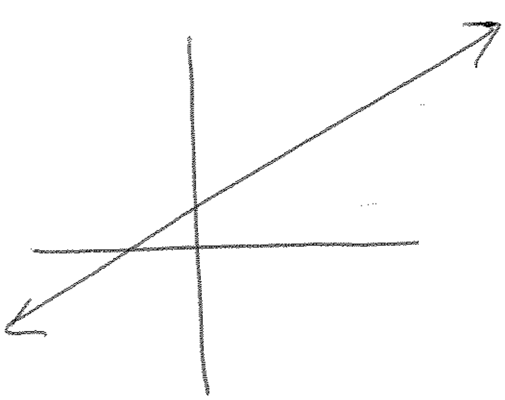
Math 1210

N1 (Polynomial Calculus)

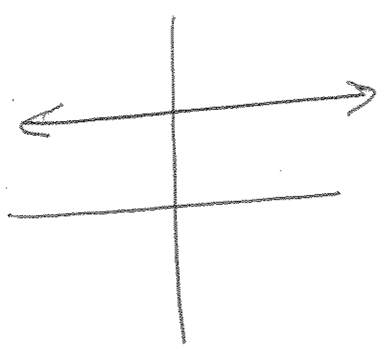
- There is only one line between any 2 pts.
- slope of a line $\Rightarrow$ the steepness of the line; defined to be the vertical change over the horizontal change; denoted by m.

In a Cartesian coordinate system, if we have a line going through (x_1, y_1) and (x_2, y_2), then the slope is

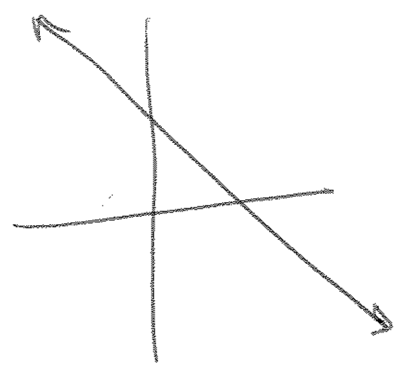
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$



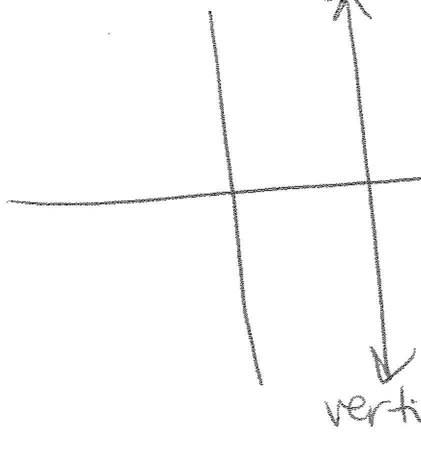
positive slope



zero slope (horizontal line)



negative slope



undefined slope (or ∞ slope)

vertical line

N1 (continued)

Ex Find the slope of the line that goes through $(-1,3)$ and $(2,4)$.

Point-Slope Form of a Line

Given $m =$ slope of my line and I know it goes through (x_1, y_1), then we know

$m = \dfrac{y_1 - y}{x_1 - x}$ for every point (x,y) on my line.

$\iff (x_1 - x)m = y_1 - y$

$\iff (x - x_1)m = y - y_1$

OR $\boxed{y - y_1 = m(x - x_1)}$

Slope-Intercept form of a Line

Given the slope m + the y-intercept $(0,b)$, the equation of the line is

$y - b = m(x - 0) \iff \boxed{y = mx + b}$

N1 (continued)

Ex Find the equation of the line going through $(-4, 1)$ and $(5, 2)$.

EX Find the equation of the line with slope $m = 3$ and y-intercept $(0, 5)$.

General Eqn of a Line

Every line can be written in the form
$$Ax + By + C = 0$$
where $A, B, C \in \mathbb{R}$.

A non-vertical line which can be written as
$$y = mx + b \iff -mx + y - b = 0$$
, so it fits the general form.

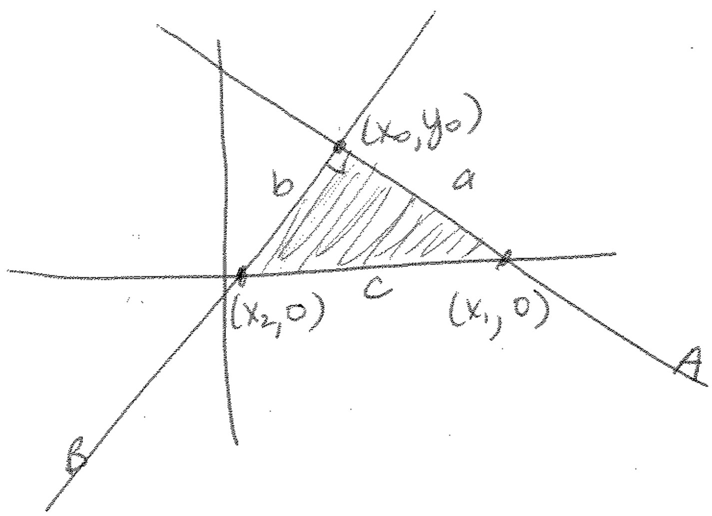
A vertical line is written as $x = k$ where k = constant.

N1 (continued)

Parallel and Perpendicular lines

① parallel lines have the same slope

② Perpendicular lines.



If lines A + B are perpendicular, then the shaded triangle is a right triangle. We can use Pythagoream Thm, $a^2+b^2=c^2$.

$$a^2 = \left(\sqrt{(x_0-x_1)^2+(y_0-0)^2}\right)^2 = (x_0-x_1)^2+y_0^2$$

$$b^2 = \left(\sqrt{(x_0-x_2)^2+(y_0-0)^2}\right)^2 = (x_0-x_2)^2+y_0^2$$

and $c^2 = \left(\sqrt{(x_1-x_2)^2+(0-0)^2}\right)^2 = (x_1-x_2)^2$

$$\Rightarrow a^2+b^2 = (x_0-x_1)^2+y_0^2+(x_0-x_2)^2+y_0^2 = (x_1-x_2)^2 = c^2$$

$$\Leftrightarrow x_0^2-2x_0x_1+x_1^2+y_0^2+x_0^2-2x_0x_2+x_2^2+y_0^2$$
$$= x_1^2-2x_1x_2+x_2^2$$

N1 (continued)

$$\Leftrightarrow \quad 2x_0^2 + \cancel{x_1^2} + \cancel{x_2^2} + 2y_0^2 - 2x_0x_1 - 2x_0x_2 = \cancel{x_1^2} - 2x_1x_2 + \cancel{x_2^2}$$

$$\Leftrightarrow \quad 2x_0^2 + 2y_0^2 - 2x_0x_1 - 2x_0x_2 = -2x_1x_2$$

$$\Leftrightarrow \quad x_0^2 + y_0^2 - x_0x_1 - x_0x_2 = -x_1x_2$$

$$\Leftrightarrow \quad y_0^2 = -x_0^2 + x_0x_1 + x_0x_2 - x_1x_2$$

$$y_0^2 = -x_0(x_0 - x_1) + x_2(x_0 - x_1)$$

$$y_0^2 = (-x_0 + x_2)(x_0 - x_1)$$

$$y_0^2 = (x_2 - x_0)(x_0 - x_1)$$

$$\frac{y_0}{x_2 - x_0} = \frac{x_0 - x_1}{y_0}$$

and

m_A = Slope of line $A = \dfrac{y_0 - 0}{x_0 - x_1} = \dfrac{y_0}{x_0 - x_1}$

m_B = Slope of line $B = \dfrac{y_0 - 0}{x_0 - x_2} = \dfrac{y_0}{x_0 - x_2}$

$$\Rightarrow \quad -m_B = \frac{1}{m_A} \quad \text{OR} \quad \boxed{m_B = \frac{-1}{m_A}}$$

So, the slopes of $\perp$ lines are negative reciprocals of one another!

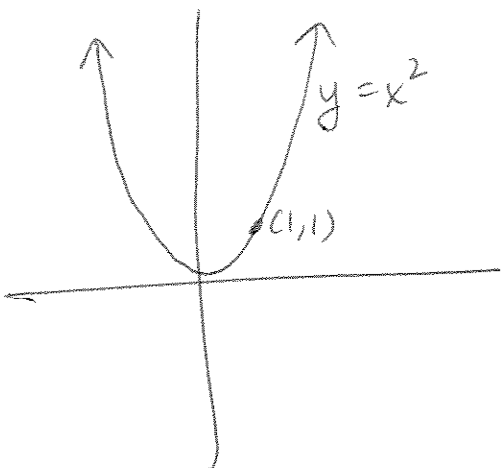
Math 1210

N1 (continued)

Ex Find the equation of the line perpendicular to $3x-4y=8$ and that passes through the point $(1, 3)$.

N2 Slope of a Curve

We know about slope of a line, but what about curves which don't have the same steepness everywhere?



Slope to left of origin:

Slope to right of origin:

Try to find the slope of the curve at the pt $(1,1)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y_2 - 1}{x_2 - 1}$$

2nd pt	slope
$(3, 9)$	
$(2, 4)$	
$(1.1, 1.21)$	
$(1.01, 1.0201)$	
$(1+h, (1+h)^2)$	

N2 (continued)

Ex Find the slope of the curve

$y = x^2 - 5x$ at $(2, -6)$.

Calculate slope between $(2, -6)$ and

$(2+h, (2+h)^2 - 5(2+h))$

N2 (continued)

Defn The slope of a function f at $(x, f(x))$ is given by $m = f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$f'(x)$ is called the derivative of f with respect to x.

other names for $f'(x)$ = slope
instantaneous rate of change
speed
velocity

Ex Find the derivative of $f(x) = 4x - 1$.

N3 Derivative of a Polynomial

Let's use our defn for a derivative to find the derivative of monomial functions.

$f(x)$	$f'(x)$
1	
x	
x^2	
x^3	
x^4	

For $f(x)=1$, $f'(x)=\lim_{h\to 0} \frac{1-1}{h} = 0$

For $f(x)=x$,

$$f'(x) = \lim_{h\to 0} \frac{(x+h)-x}{h} = 1$$

For $f(x)=x^2$

$$f'(x)= \lim_{h\to 0} \frac{(x+h)^2 - x^2}{h}$$

$$=$$

For $f(x)=x^3$, $f'(x)=$

For $x^4=f(x)$, $f'(x)=$

N3 (continued)

What is our pattern?

$\boxed{\underline{\text{Thm A}}\quad \text{If } n \in \mathbb{Z}^+, \text{ then } (x^n)' = n x^{n-1}}$ The Power Rule

★ Other derivative notation $\dfrac{d(x^n)}{dx} = (x^n)'$.

Thm B The derivative of a constant times a function is that constant times the derivative of the function. And the derivative of a sum (or difference) of functions is the sum (or difference) of the derivatives of the functions.

i.e. $(cf)' = cf'$ and $(f+g)' = f' + g'$

Ex 1 Find the derivative of

$y = -2x^2 + 5x - 1$

Ex 2 Find the derivative of

$f = 4x^8 - 3x^5 + 2x^2 + 7x$

N3 (continued)

Ex 3 Find the slope of $y = 4x^3 - 3x^2 + 9x$

when $x = 1$.

Ex 4 Find the rate of change of $f(x) = -x^3 + 5x$

$\underbrace{\text{with respect to}}_{\text{(wrt)}}$ x when $x = -1$.

Velocity + Acceleration

velocity = how fast distance changes over time

= rate of change of distance wrt time

$= \frac{ds}{dt} = s'$ (if s = distance or position)

acceleration = how velocity changes over time

$= \frac{dv}{dt} = v'$

N3 (continued)

(#13) A ball is thrown straight up + follows the path given by

$$s(t) = -16t^2 + 32t + 6 \qquad t \text{ in seconds}$$

generic shape of curve

velocity = ?   acceleration = ?

at time t

How high does the ball go?

N4 (Antiderivatives of Polynomials)

$\boxed{\text{\underline{Defn} If } f(x) \text{ is a function, then an antiderivative for } f \text{ is a function having } f(x) \text{ as its derivative.}}$

- An antiderivative "undoes" the derivative.
- For any function, there are infinitely many, or a "family," of antiderivatives.

We know $\left(x^{n+1}\right)' = (n+1)x^n$

$\Longleftrightarrow$

$\Longleftrightarrow$

Defn B If $f(x)$ is a function, the set of all antiderivatives for $f(x)$ is denoted by $\int f(x)\,dx$ + is called the indefinite integral of $f(x)$.

$$\Rightarrow \int x^n\,dx = \frac{x^{n+1}}{n+1} + C$$ where C = arbitrary constant

$\forall\, n \in \mathbb{Z}^+$

Power Rule

N4 (continued)

Thm B Integral operator is linear, i.e.

(1) $\int a f(x)\,dx = a\int f(x)\,dx$

and

(2) $\int (f(x)+g(x))\,dx = \int f(x)\,dx + \int g(x)\,dx$

if f + g are functions and $a \in \mathbb{R}$.

Ex 1 $\int (x^2 - 4x + 7)\,dx$

Ex2 $F = \int 2x^3 - 5x^2\,dx = ?$ when we know $F = -4$ when $x = 0$.

N4 (continued)

Ex 3 The acceleration for an object due to gravity is -32 ft/sec². A ball is thrown straight up with an initial velocity of 25 ft/sec, after which the only force acting on the ball is gravity. What is the velocity t seconds later? When does it reach its max ht?

$$a(t) = v'(t) \Rightarrow$$

N4 (continued)

Ex 4 Refer to example 3, except this time the ball was thrown from an initial height of 6 ft. Find the height of the ball at any time t. What is the max ht?