Section	Key Terms	Formula
10.5	Asymptotes Horizontal: $y = b$	$\lim_{x\to+\infty} f(x) = b$ or $\lim_{x\to-\infty} f(x) = b$
	Vertical: $x = c$ for rational function $y = f(x)/g(x)$	y unbounded near $x = c$ if $g(c) = 0$ and $f(c) \neq 0$

REVIEW EXERCISES *Additional Practice with Guided Solutions on CD-ROM*

Section 10.1

In Problems 1–4, find all critical points and determine whether they are relative maxima, relative minima, or horizontal points of inflection.

1. $y = -x^2$
2. $p = q^2 - 4q - 5$
3. $f(x) = 1 - 3x + 3x^2 - x^3$
4. $f(x) = \dfrac{3x}{x^2 + 1}$

In Problems 5–10.

(a) Find all critical values, including those where $f'(x)$ is undefined.
(b) Find the relative maxima and minima, if any exist.
(c) Find the horizontal points of inflection, if any exist.
(d) Sketch the graph.

5. $y = x^3 + x^2 - x - 1$
6. $f(x) = 4x^3 - x^4$
7. $f(x) = x^3 - \dfrac{15}{2}x^2 - 18x + \dfrac{3}{2}$
8. $y = 5x^7 - 7x^5 - 1$
9. $y = x^{2/3} - 1$
10. $y = x^{2/3}(x - 4)^2$

Section 10.2

11. Is the graph of $y = x^4 - 3x^3 + 2x - 1$ concave up or concave down at $x = 2$?
12. Find intervals where the graph of $y = x^4 - 2x^3 - 12x^2 + 6$ is concave upward and intervals where it is concave downward, and find points of inflection.
13. Find the relative maxima, relative minima, and points of inflection of the graph of $y = x^3 - 3x^2 - 9x + 10$.

In Problems 14 and 15, find any relative maxima, relative minima, and points of inflection, and sketch each graph.

14. $y = x^3 - 12x$
15. $y = 2 + 5x^3 - 3x^5$

Section 10.3

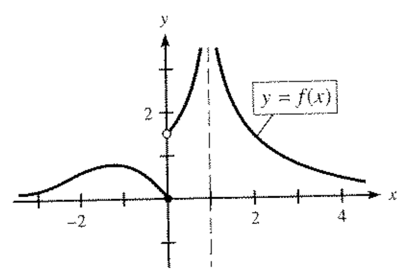
16. Given $R = 280x - x^2$, find the absolute maximum and minimum for R when (a) $0 \le x \le 200$ and (b) $0 \le x \le 100$.
17. Given $y = 6400x - 18x^2 - \dfrac{x^3}{3}$, find the absolute maximum and minimum for y when (a) $0 \le x \le 50$ and (b) $0 \le x \le 100$.

Section 10.5

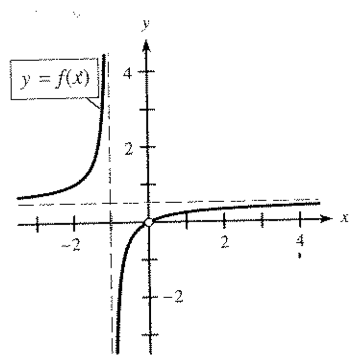
In Problems 18 and 19, use the graphs to find the following items.

(a) vertical asymptotes **(b) horizontal asymptotes**
(c) $\lim_{x\to+\infty} f(x)$ **(d) $\lim_{x\to-\infty} f(x)$**

18.



19.



In Problems 20 and 21, find any horizontal asymptotes and any vertical asymptotes.

20. $y = \dfrac{3x + 2}{2x - 4}$ 21. $y = \dfrac{x^2}{1 - x^2}$

In Problems 22–24:
(a) Find any horizontal and vertical asymptotes.
(b) Find any relative maxima and minima.
(c) Sketch each graph.

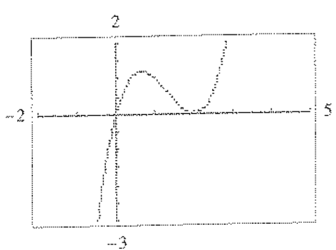
22. $y = \dfrac{3x}{x + 2}$ 23. $y = \dfrac{8(x - 2)}{x^2}$

24. $y = \dfrac{x^2}{x - 1}$

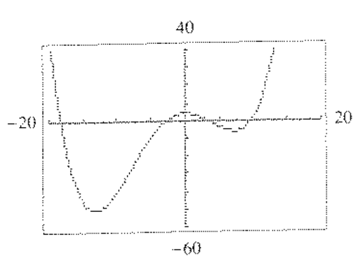
Sections 10.1 and 10.2

In Problems 25 and 26, a function and its graph are given.
(a) Use the graph to determine (estimate) x-values where $f'(x) > 0$, where $f'(x) < 0$, and where $f'(x) = 0$.
(b) Use the graph to determine x-values where $f''(x) > 0$, where $f''(x) < 0$, and where $f''(x) = 0$.
(c) Check your conclusions to (a) by finding $f'(x)$ and graphing it with a graphing utility.
(d) Check your conclusions to (b) by finding $f''(x)$ and graphing it with a graphing utility.

25. $f(x) = x^3 - 4x^2 + 4x$

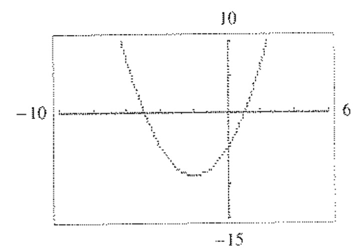


26. $f(x) = 0.0025x^4 + 0.02x^3 - 0.48x^2 + 0.08x + 4$

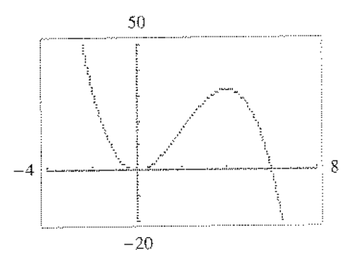


In Problems 27 and 28, $f'(x)$ and its graph are given.
(a) Use the graph of $f'(x)$ to determine (estimate) where the graph of $f(x)$ is increasing, where it is decreasing, and where it has relative extrema.
(b) Use the graph of $f'(x)$ to determine where $f''(x) > 0$, where $f''(x) < 0$, and where $f''(x) = 0$.
(c) Verify that the given $f(x)$ has $f'(x)$ as its derivative, and graph $f(x)$ to check your conclusions in (a).
(d) Calculate $f''(x)$ and graph it to check your conclusions in (b).

27. $f'(x) = x^2 + 4x - 5 \quad \left(\text{for } f(x) = \dfrac{x^3}{3} + 2x^2 - 5x\right)$

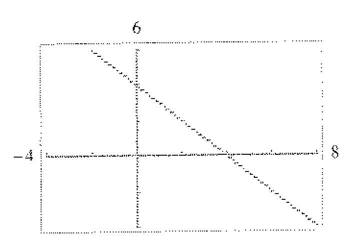


28. $f'(x) = 6x^2 - x^3 \quad \left(\text{for } f(x) = 2x^3 - \dfrac{x^4}{4}\right)$

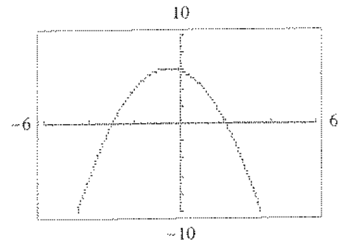


In Problems 29 and 30, $f''(x)$ and its graph are given.
(a) Use the graph to determine (estimate) where the graph of $f(x)$ is concave upward, where it is concave downward, and where it has points of inflection.
(b) Verify that the given $f(x)$ has $f''(x)$ as its second derivative, and graph $f(x)$ to check your conclusions in (a).

29. $f''(x) = 4 - x \quad \left(\text{for } f(x) = 2x^2 - \dfrac{x^3}{6}\right)$



30. $f''(x) = 6 - x - x^2$ $\left(\text{for } f(x) = 3x^2 - \dfrac{x^3}{6} - \dfrac{x^4}{12}\right)$



APPLICATIONS

Sections 10.1–10.3

In Problems 31–36, cost, revenue, and profit are in dollars and x is the number of units.

31. ***Cost*** Suppose the total cost function for a product is

$$C(x) = 3x^2 + 15x + 75$$

How many units minimize the average cost? Find the minimum average cost.

32. ***Revenue*** Suppose the total revenue function for a product is given by

$$R(x) = 32x - 0.01x^2$$

(a) How many units will maximize the total revenue? Find the maximum revenue.
(b) If production is limited to 2500 units, how many units will maximize the total revenue? Find the maximum revenue.

33. ***Profit*** Suppose the profit function for a product is

$$P(x) = 1080x + 9.6x^2 - 0.1x^3 - 50{,}000$$

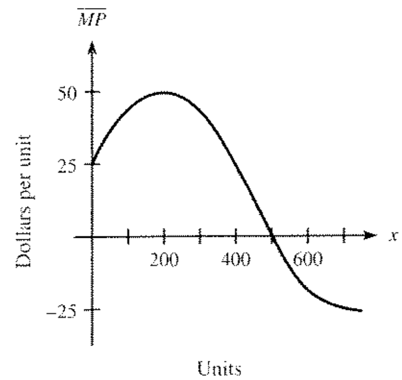
Find the maximum profit.

34. ***Profit*** How many units (x) will maximize profit if $R(x) = 46x - x^2$ and $C(x) = 5x^2 + 10x + 3$?

35. ***Profit*** A product can be produced at a total cost $C(x) = 800 + 4x$, where x is the number produced and is limited to at most 150 units. If the total revenue is given by $R(x) = 80x - \frac{1}{4}x^2$, determine the level of production that will maximize the profit.

36. ***Average cost*** The total cost function for a product is $C = 2x^2 + 54x + 98$. Producing how many units will minimize average cost?

37. ***Marginal profit*** The following figure shows the graph of a marginal profit function for a company. At what level of sales will profit be maximized? Explain.



38. ***Productivity—diminishing returns*** Suppose the productivity P of an individual worker (in number of items produced per hour) is a function of the number of hours of training t according to

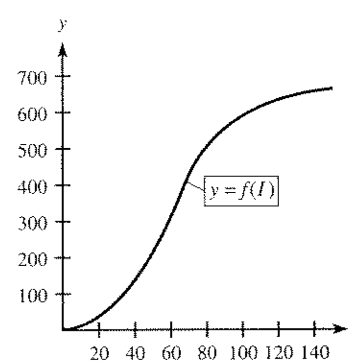
$$P(t) = 5 + \frac{95t^2}{t^2 + 2700}$$

Find the number of hours of training at which the rate of change of productivity is maximized. (That is, find the point of diminishing returns.)

39. ***Output*** The figure shows a typical graph of output y (in thousands of dollars) as a function of capital investment I (also in thousands of dollars).
(a) Is the point of diminishing returns closest to when $I = 20$, when $I = 60$, or when $I = 120$? Explain.
(b) The average output per dollar of capital investment is defined as the total output divided by the amount of capital investment; that is,

$$\text{Average output} = \frac{f(I)}{I}$$

Calculate the slope of a line from (0, 0) to an arbitrary point $(I, f(I))$ on the output graph. How is this slope related to the average output?
(c) Is the maximum average output attained when the capital investment is closest to $I = 40$, to $I = 70$, or to $I = 140$? Explain.



40. ***Revenue*** MMR II Extreme Bike Shop sells 54 basic-style mountain bikes per month at a price of \$385 each. Market research indicates that MMR II could sell 10 more of these bikes if the price were \$25 lower. At what selling price will MMR II maximize the revenue from these bikes?

41. ***Profit*** If in Problem 40 the mountain bikes cost the shop \$200 each, at what selling price will MMR II's profit be a maximum?

42. ***Profit*** Suppose that for a product in a competitive market, the demand function is $p = 1200 - 2x$ and the supply function is $p = 200 + 2x$, where x is the number of units and p is in dollars. A firm's average cost function for this product is

$$\overline{C}(x) = \frac{12{,}000}{x} + 50 + x$$

Find the maximum profit. *Hint:* First find the equilibrium price.

43. ***Profit*** The monthly demand function for x units of a product sold at \$$p$ per unit by a monopoly is $p = 800 - x$, and its average cost is $\overline{C} = 200 + x$.
(a) Determine the quantity that will maximize profit.
(b) Find the selling price at the optimal quantity.

44. ***Profit*** Suppose that in a monopolistic market, the demand function for a commodity is

$$p = 7000 - 10x - \frac{x^2}{3}$$

where x is the number of units and p is in dollars. If a company's average cost function for this commodity is

$$\overline{C}(x) = \frac{40{,}000}{x} + 600 + 8x$$

find the maximum profit.

Section 10.4

45. ***Reaction to a drug*** The reaction R to an injection of a drug is related to the dosage x (in milligrams) according to

$$R(x) = x^2\left(500 - \frac{x}{3}\right)$$

Find the dosage that yields the maximum reaction.

46. ***Productivity*** The number of parts produced per hour by a worker is given by

$$N = 4 + 3t^2 - t^3$$

where t is the number of hours on the job without a break. If the worker starts at 8 A.M., when will she be at maximum productivity during the morning?

47. ***Population*** Population estimates show that the equation $P = 300 + 10t - t^2$ represents the size of the graduating class of a high school, where t represents the number of years after 2000, $0 \le t \le 10$. What will be the largest graduating class in the decade?

48. ***Night brightness*** Suppose that an observatory is to be built between cities A and B, which are 30 miles apart. For the best viewing, the observatory should be located where the night brightness from these cities is minimum. If the night brightness of city A is 8 times that of city B, then the night brightness b between the two cities and x miles from A is given by

$$b = \frac{8k}{x^2} + \frac{k}{(30 - x)^2}$$

where k is a constant. Find the best location for the observatory; that is, find x that minimizes b.

49. ***Product design*** A playpen manufacturer wants to make a rectangular enclosure with maximum play area. To remain competitive, he wants the perimeter of the base to be only 16 feet. What dimensions should the playpen have?

50. ***Printing design*** A printed page is to contain 56 square inches and have a $\frac{3}{4}$-inch margin at the bottom and 1-inch margins at the top and on both sides. Find the dimensions that minimize the size of the page (and hence the costs for paper).

51. ***Drug sensitivity*** The reaction R to an injection of a drug is related to the dosage x, in milligrams, according to

$$R(x) = x^2\left(500 - \frac{x}{3}\right)$$

The sensitivity to the drug is defined by dR/dx. Find the dosage that maximizes sensitivity.

52. ***Photosynthesis*** The amount of photosynthesis that takes place in a certain plant depends on the intensity of light x, in lumens, according to the equation

$$f(x) = 145x^2 - 30x^3$$

The rate of change of the amount of photosynthesis with respect to the intensity is $f'(x)$. Find the intensity that maximizes the rate of change.