7.1 Area Between Curves

Theorem 1: Area Between 2 Curves

If \( f(x) \) and \( g(x) \) are continuous and \( f(x) \geq g(x) \) on \([a, b]\), then the area bounded by \( y = f(x) \) and \( y = g(x) \) for \( a \leq x \leq b \) is given exactly by

\[
A = \int_a^b [f(x) - g(x)] \, dx
\]

Example 1: Find the area bounded by \( y = x^2 + 2 \), \( y = 0 \) on \( 0 \leq x \leq 3 \).
7.1 (cont)

Ex 2 Find the area of the region bounded by the given curves over the given x-interval.

(a) \( y = 2x + 6 \), \( y = 3 \), \(-1 \leq x \leq 2\)

(b) \( y = \frac{1}{x} \), \( y = e^{-x} \), \( \frac{1}{2} \leq x \leq 1 \)
Ex 3 Find the area bounded by the given curves.

(a) $y = x^3 + 1$, $y = x + 1$

(b) $y = x^3 - 6x^2 + 9x$, $y = x$
8.1 Functions of Several Variables

\[ z = f(x, y) \]

- \( x, y \) input variables; independent
- \( z \) output variable; dependent on \( x \) + \( y \)
- graphs in 3d
- domain = still the set of allowable inputs \((x + y)\)
- range = still set of possible outputs \(z\)

Ex 1 Find \( f(5, 6) \) for \( f(x, y) = 2xy - 7x^2 + y - 5 \)

Ex 2 Find \( p(2, 2) \) for \( p(x, y) = -x^2 + 2xy - 2y^2 - 4x + 12y + 5 \)
Ex 3  Find \(N(x, 2x)\) for \(N(x, y) = 3xy + x^2 - y^2 + 1\)

Ex 4  Find \(\frac{f(x, y+k) - f(x, y)}{k}\) for \(f(x, y) = x^2 + 2y^2\).
8.2 Partial Derivatives

For $z = f(x, y)$, we can’t talk about “the” derivative, since there are 2 independent variables. We use partial derivatives instead.

\[
\frac{df}{dx} = \text{partial derivative of } f \text{ wrt } x \\
\frac{df}{dy} = \text{ partial derivative of } f \text{ wrt } y
\]

When taking \( \frac{df}{dx} \), treat \( y \) as a constant.

\[
\text{When taking } \frac{df}{dy}, \text{ treat } x \text{ as a constant.}
\]

Ex 1: Find \( f_x + f_y \) for \( f(x, y) = 6x^2 - 8xy + 3y^2 - 1 \)
Ex 2 Find \( \frac{\partial^2 f}{\partial y} \) for \( f(x,y) = g = (2x^2y - \frac{3}{y})^8 \)

Ex 3 Find \( f_x(1,-1) \) for \( f(x,y) = \frac{2xy}{1 + x^2y^2} \)
8.2 (cont)

Second Partial Derivatives

\[ \frac{\partial^2 f}{\partial x^2} = f_{xx}, \quad \frac{\partial^2 f}{\partial y \partial x} = f_{yx}, \quad \frac{\partial^2 f}{\partial x \partial y} = f_{xy}, \quad \frac{\partial^2 f}{\partial y^2} = f_{yy} \]

Ex 4 Find \( f_{yx}, f_{xy}, f_{yy} \) + \( f_{xx} \) for \( f(x,y) = -4xy^3 + 9x^2y^2 \)
8.2 (cont)

Example 5

For \( S(x,y) = x^3 \ln y + 4y^2e^x \), find

(a) \( S_x \) at \((1, 1)\)

(b) \( S_{xy} \) at \((-1, 1)\)