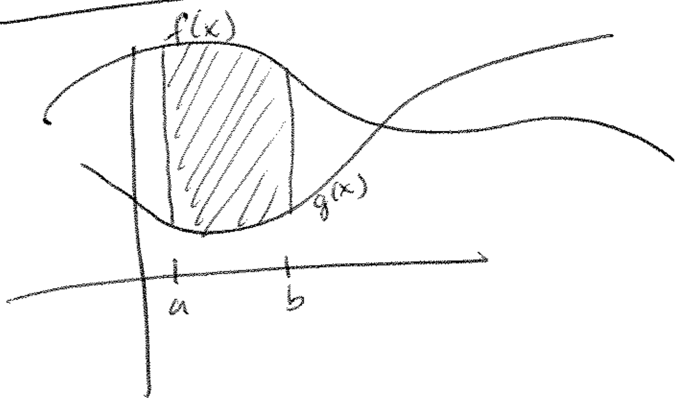
7.1 Area Between Curves

Thm 1 Area Between 2 curves

If $f(x)$ + $g(x)$ are continuous + $f(x) \geq g(x)$ on $[a,b]$, then the area bounded by $y=f(x)$ + $y=g(x)$ for $a \leq x \leq b$ is given exactly by

$$A = \int_a^b [f(x) - g(x)]\,dx$$



Ex 1 Find the area bounded by $y = x^2 + 2$, $y = 0$ on $0 \leq x \leq 3$.

7.1 (cont)

Ex 2 Find the area of the region bounded by the given curves over the given x-interval.

(a) $y = 2x+6$, $y = 3$, $-1 \leq x \leq 2$

(b) $y = 1/x$, $y = e^{-x}$, $1/2 \leq x \leq 1$

7.1 (cont)

Ex 3 Find the area bounded by the given curves.

(a) $y = x^3 + 1, \quad y = x + 1$

(b) $y = x^3 - 6x^2 + 9x, \quad y = x$

8.1 Functions of Several Variables

$z = f(x, y)$

- x, y input variables; independent
- z output variable; dependent on $x + y$
- graphs in 3d
- domain ⇒ still the set of allowable inputs $(x + y)$
- range ⇒ still set of possible outputs (z)

<u>Ex 1</u> Find $f(5,6)$ for $f(x,y) = 2xy - 7x^2 + y - 5$

<u>Ex 2</u> Find $P(2,2)$ for $P(x,y) = -x^2 + 2xy - 2y^2 - 4x + 12y + 5$

8.1 (cont)

Ex 3 Find $N(x, 2x)$ for $N(x,y) = 3xy + x^2 - y^2 + 1$

Ex 4 Find $\dfrac{f(x, y+k) - f(x,y)}{k}$ for $f(x,y) = x^2 + 2y^2$.

8.2 Partial Derivatives

for $z = f(x,y)$, we can't talk about "the" derivative, since there are 2 independent variables. We use partial derivatives instead.

$\frac{\partial f}{\partial x}$ = partial derivative of f wrt x

$\frac{\partial f}{\partial y}$ = " " " wrt y

Notation

$\frac{\partial f}{\partial x} = f_x$

$\frac{\partial f}{\partial y} = f_y$

When taking $\frac{\partial f}{\partial x}$, treat y as a constant.

" " $\frac{\partial f}{\partial y}$, " x " " " .

Ex 1 Find f_x + f_y for $f(x,y) = 6x^2 - 8xy + 3y^2 - 1$

8.2 (cont)

Ex 2 Find $\frac{\partial z}{\partial y}$ for $f(x,y) = z = \left(2x^2y - \frac{3}{y}\right)^8$

Ex 3 Find $f_x(1,-1)$ for $f(x,y) = \frac{2xy}{1+x^2y^2}$

Second Partial Derivatives

$$\frac{\partial^2 f}{\partial x^2} = f_{xx}, \quad \frac{\partial^2 f}{\partial x \partial y} = f_{yx}, \quad \frac{\partial^2 f}{\partial y \partial x} = f_{xy}, \quad \frac{\partial^2 f}{\partial y^2} = f_{yy}$$

Ex 4 Find f_{yx}, f_{xy}, f_{yy} + f_{xx} for $f(x,y) = -4x^3y^5 + 9x^6y^2$

8.2 (cont)

Ex 5 For $S(x,y) = x^3 \ln y + 4y^2 e^x$, find

(a) $S_x(-1, 1)$

(b) $S_{xy}(-1, 1)$