5.1 First Derivative and Graphs

**Critical Values**

The values of $x$ in the domain of $f(x)$ where $f'(x) = 0$ or $f'(x)$ dne.

Then if $f(x)$ is continuous on $(a,b)$, $c \in (a,b)$, and $f(c)$ is a local extrema, then either $f'(c) = 0$ or $f'(c)$ dne.

**Local Extrema =** local min/max pts

We'll use 1st derivative to find min/max pts. *(nice pictures top of pg 282)*

$f'(x) = 0$

$f'(x)$ dne
5.1 (cont.)

Ex 1 Find intervals where \( f(x) \) increasing and decreasing and find min/max pts.

(a) \( f(x) = -3x^2 + 12x - 5 \)

(b) \( f(x) = x \ln x - x \)
5.1 (cont)

Ex 2 Find min/max pts, increasing, decreasing regions for \( f(x) \) and sketch graph.

(a) \( f(x) = -x^4 + 50x^2 \)

(b) \( f(x) = x^3 - 12x + 2 \)
EX 3 Given this graph of $f'(x)$, sketch a possible graph of $f(x)$.

EX 4 Given graph of $f(x)$, sketch possible graph of $f'(x)$.
5.1 (cont)

**Exs** Find all min/max pts.

\[ f(x) = 6(4 - x)^{2/3} + 4 \]
5.2 Second Derivative & Graphs

**Defn: Concavity**

The graph of \( f(x) \) is concave upward on \((a,b)\) if \( f''(x) \) is increasing on \((a,b)\) and concave downward on \((a,b)\) if \( f''(x) \) is decreasing on \((a,b)\).

**Second Derivative**

\[ f''(x), \quad \frac{d^2f}{dx^2}, \quad D_x^2(f) \quad \text{all the notation for second derivative} \]

It's the derivative of the first derivative.

\[
\begin{align*}
\Rightarrow \text{ If } f''(x) > 0, & \text{ concave up.} \\
\text{If } f''(x) < 0, & \text{ concave down.} \\
\text{If } f''(x) = 0, & \text{ it may be an inflection pt.} \\
\end{align*}
\]

An inflection pt is where the concavity changes.

**Thm: Inflection Points**

If \( y = f(x) \) is continuous on \((a,b)\) and has an inflection pt at \( x = c \), then either \( f''(c) = 0 \) or \( f''(c) \) does not exist.
5.2 (cont)

Ex 1 Use this graph to identify
(a) intervals where $g$ concave up, $g''(x) > 0$, $g'(x)$ increasing
(b) intervals where $g$ concave down, $g''(x) < 0$, $g'(x)$ decreasing
(c) inflection pts
(d) min/max pts

Ex 2
Find $f''(x)$ for $f(x) = x^3 - 24x^{4/3}$
Ex 3. Find $y''$ for $y = x^2 \ln x$

Ex 4. Find where $f(x)$ is concave up, concave down, and inflection points.

$$f(x) = x^4 - 2x^3 - 3x + 12$$
5.2 (cont)

Ex 5  Analyze graph of \( f(x) = 3x^5 - 5x^4 + \text{graph.} \)
5.4 Curve-Sketching Techniques

Graphing Strategy

1. (a) domain of \( f(x) \)
   (b) VA and HA
   (c) x-intercepts (optional)
2. \( f'(x) \) sign line/min/max pts
3. \( f''(x) \) sign line/inflection pts
4. sketch graph.

Ex 1: Analyze graph of \( f(x) = \frac{2x-4}{x+2} \)
5.4 (cont)

Ex 2: Analyze/ sketch graph of

(a) $f(x) = \ln(x^2+4)$

(b) $f(x) = \frac{x^2 - 5x - 6}{x^2}$
Ex. 3  Analyze/sketch graph of \( f(x) = e^{-2x^2} \)

Ex. 4  Analyze/sketch graph of \( f(x) = x - \frac{16}{x^2} \)

(show \( y=x \) is oblique asymptote.)
5.5 Absolute Maxima and Minima

**Defn**
If \( f(c) \geq f(x) \) \( \forall x \in \text{domain of } f \), then \( f(c) \) is called absolute maximum value of \( f(x) \).

If \( f(c) \leq f(x) \) \( \forall x \in \text{domain of } f \), then \( f(c) \) is called absolute minimum value of \( f(x) \).

**Thm**
If \( f(x) \) is continuous on \([a,b]\), then it has both a maximum and minimum pt there.

We find abs. max/min pts by looking at:
1. pts where \( f'(x) = 0 \) (stationary pts)
2. pts where \( f'(x) \) does not exist (singular pts)
3. endpoints

**Second Derivative Test**
If \( x=c \) is a critical value, then
- if \( f''(c) > 0 \), there is min pt at \( x=c \).
- if \( f''(c) < 0 \), there is max pt at \( x=c \).
Ex 1 Find absolute min/max pts.

(a) \( f(x) = x^4 - 4x^3 \)

(b) \( f(x) = \frac{9-x^2}{x^2+4} \)
Ex 2  Find absolute min on \((0, \infty)\) for 
\[ f(x) = (2-x)(x+1)^2. \]

Ex 3  Find absolute min/max pts for 
\[ f(x) = 2x^3 - 3x^2 - 12x + 24. \]

(a) on \([-2, 1]\)

(b) on \([-2, 3]\)
Ex 4 Find abs. min/max pts (if they exist) for \( f(x) = \frac{e^x}{x} \) on \((0, \infty)\).
5.6 Optimization (Story Problem II)

Ex. 1: What quantity should be added to $x$ or subtracted from $x$ to produce the maximum product of the results?

Strategy
1. Write down info, assign variable name, draw picture (if applicable).
2. Find primary eqn (the one we need to maximize or minimize).
3. We may need to use a secondary eqn to get rid of a variable in primary eqn.
4. Take derivative of primary eqn, set it equal to 0, solve.
5. Make sure point we get is really a max or min.
S.6 (cont.)

Ex 2: Find the dimensions of a rectangle with area 225 ft² that has the least perimeter. What is that perimeter?
5.6 (cont)

EX 3 A company manufactures and sells x digital cameras per week. The weekly price-demand and cost equations are
\[ p = 400 - 0.4x \quad \text{and} \quad C(x) = 2000 + 160x. \]

(a) What price should the company charge for the cameras and how many cameras should be produced to maximize revenue? What is max revenue?

(b) What is profit for? Maximize profit.
Ex 4. A 300-room hotel in Las Vegas is filled to capacity every night at $80/room. For each $1 increase in rent, 3 fewer rooms are rented. If each rented room costs $10 to service per day, how much should they charge per room to maximize profit? What is max profit?