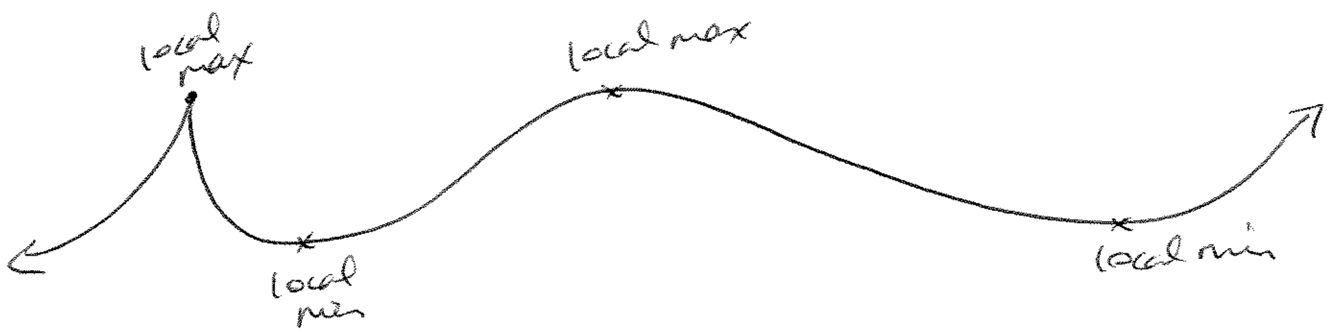
5.1 First Derivative and Graphs

Critical Values
The values of x in the domain of $f(x)$ where $f'(x)=0$ or $f'(x)$ dne.

Thm If $f(x)$ is continuous on (a,b), $c \in (a,b)$, and $f(c)$ is a local extrema, then either $f'(c)=0$ or $f'(c)$ dne.

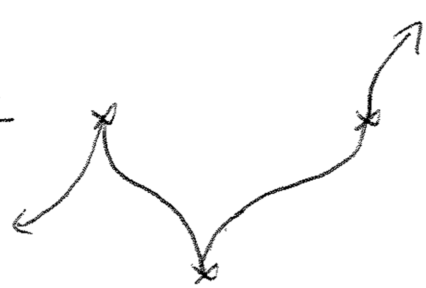
local extrema $\Rightarrow$ local min/max pts



We'll use 1st derivative to find min/max pts.
*(nice pictures top of pg 282)

$f'(x)=0$

$f'(x)$ dne



5.1 (cont)

Ex 1 Find intervals where $f(x)$ increasing + decreasing + find min/max pts.

(a) $f(x) = -3x^2 + 12x - 5$

(b) $f(x) = x\ln x - x$

5.1 (cont)

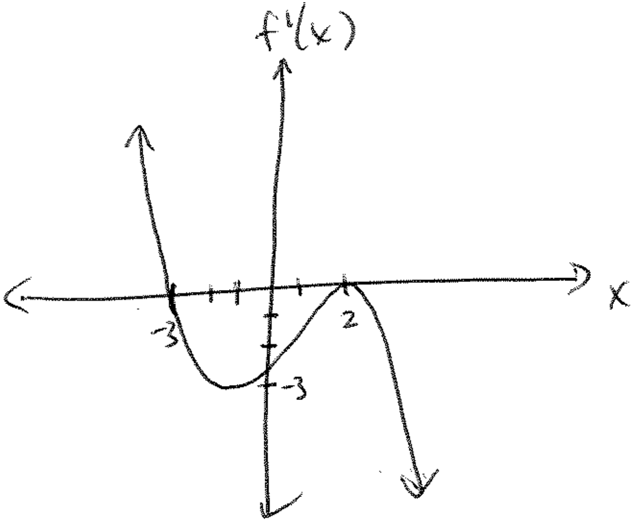
Ex 2 Find min/max pts, increasing, decreasing regions ~~for~~ f(x) + sketch graph.

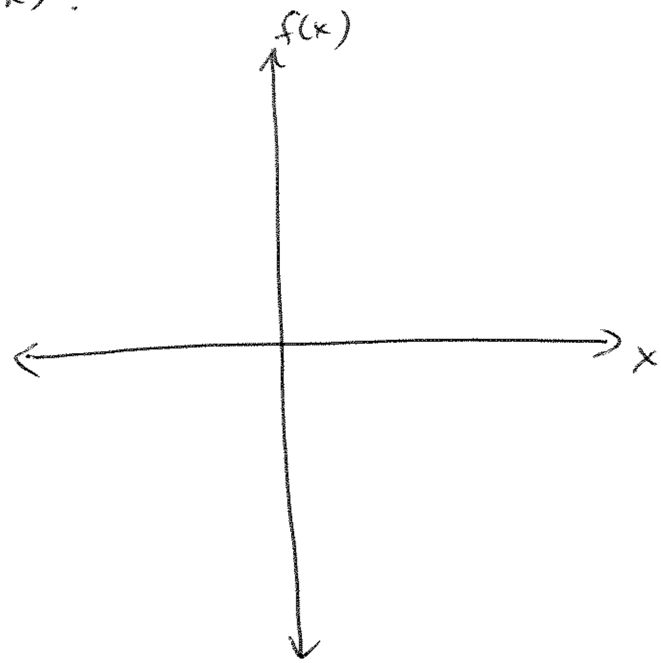
(a) $f(x) = -x^4 + 50x^2$

(b) $f(x) = x^3 - 12x + 2$

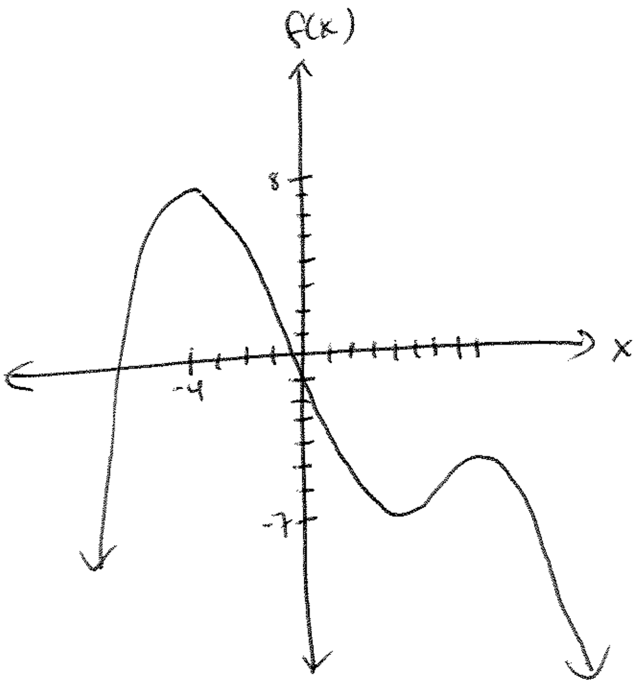
5.1 (cont)

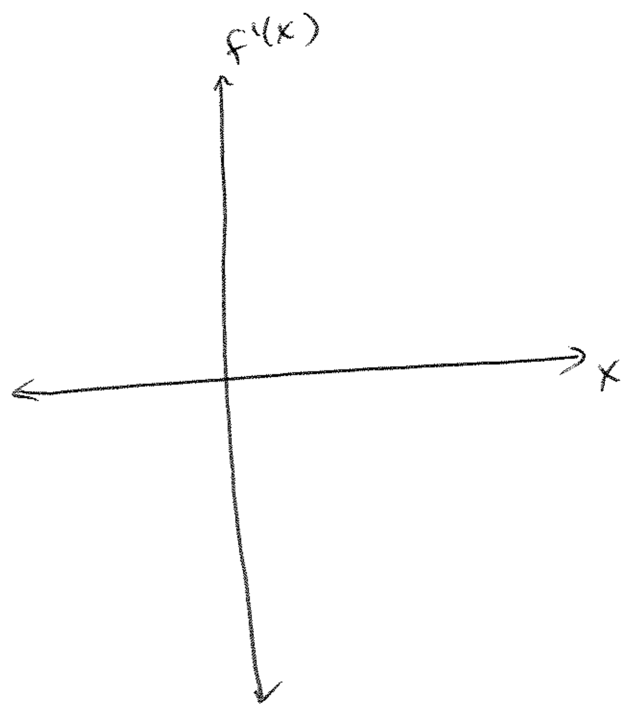
Ex 3 Given this graph of $f'(x)$, sketch a possible graph of $f(x)$.





Ex 4 Given graph of $f(x)$, sketch possible graph of $f'(x)$.





5.1 (cont)

Ex 5 Find all min/max pts.

$$f(x) = 6(4-x)^{2/3} + 4$$

5.2 Second Derivative + Graphs

Defn Concavity

The graph of $f(x)$ is concave upward on (a,b) if $f'(x)$ is increasing on (a,b) + is concave downward on (a,b) if $f'(x)$ is decreasing on (a,b).

Second Derivative

$f''(x)$, $\frac{d^2f}{dx^2}$, $D_x^2(f)$ all the notation for second derivative

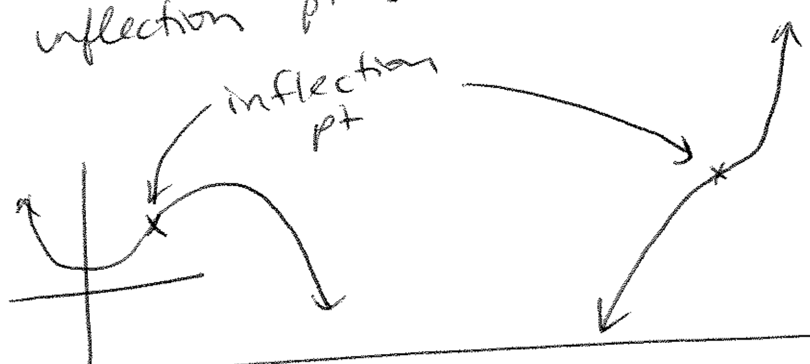
It's the derivative of the first derivative

$\Rightarrow$ If $f''(x) > 0$, concave up.

If $f''(x) < 0$, concave down.

If $f''(x) = 0$, it may be an inflection pt.

An inflection pt is where the concavity changes.



Thm Inflection Points

If $y = f(x)$ is continuous on (a,b) + has an inflection pt at $x=c$, then either $f''(c)=0$ or $f''(c)$ dne.

5.2 (cont)

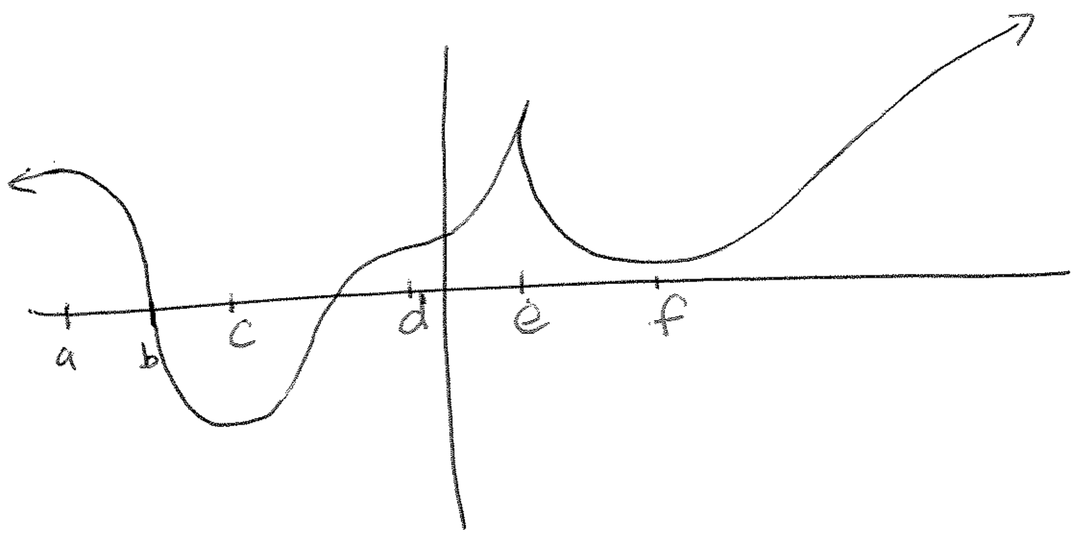
Ex 1 Use this graph to identify

(a) intervals where g concave up, $g''(x) > 0$, $g'(x)$ increasing.

(b) intervals where g concave down, $g''(x) < 0$ $g'(x)$ decreasing.

(c) inflection pts

(d) min/max pts



Ex 2
Find $f''(x)$ for $f(x) = x^3 - 24x^{1/3}$

5.2 (cont)

Ex 3 Find y'' for $y = x^2 \ln x$

Ex 4 Find where $f(x)$ is concave up, concave down + inflection pts.

$$f(x) = x^4 - 2x^3 - 36x + 12$$

5.2 (cont)

Ex 5 Analyze graph of $f(x) = 3x^5 - 5x^4$ + graph.

5.4 Curve-Sketching Techniques

Graphing Strategy

1. (a) domain of $f(x)$
   (b) VA + HA
   (c) x-intercepts (optional)
2. $f'(x)$ sign line / min/max pts
3. $f''(x)$ sign line / inflection pts
4. sketch graph.

Ex 1 Analyze graph of $f(x) = \frac{2x-4}{x+2}$

5.4 (cont)

Ex 2 Analyze / sketch graph of

(a) $f(x) = \ln(x^2+4)$

(b) $f(x) = \dfrac{x^2-5x-6}{x^2}$

5.4 (cont)

Ex 3 Analyze/sketch graph of $f(x) = e^{-2x^2}$

Ex 4 Analyze/sketch graph of $f(x) = x - \frac{16}{x^3}$.
(Show $y = x$ is oblique asymptote.)

5.5 Absolute Maxima and Minima

Defn

If $f(c) \geq f(x)$ $\forall x \in$ domain of f, then $f(c)$ is called absolute maximum value of $f(x)$.

If $f(c) \leq f(x)$ $\forall x \in$ domain of f, then $f(c)$ is called absolute minimum value of $f(x)$.

Thm

If $f(x)$ is continuous on $[a,b]$, then it has both a maximum & minimum pt there.

We find abs. max/min pts by looking at

1. pts where $f'(x)=0$ (stationary pts)
2. pts where $f'(x)$ dne (singular pts)
3. endpts

Second Derivative Test

If $x=c$ is a critical value, then

if $f''(c)>0$, there is min pt at $x=c$.

if $f''(c)<0$, there is max pt at $x=c$.

5.5 (cont)

Ex 1 Find absolute min/max pts.

(a) $f(x) = x^4 - 4x^3$

(b) $f(x) = \dfrac{9-x^2}{x^2+4}$

5.5 (cont)

Ex 2 Find absolute min on $[0,\infty)$ for
$f(x)=(2-x)(x+1)^2$.

Ex 3 Find absolute min/max pts for
$f(x)=2x^3-3x^2-12x+24$

(a) on $[-2,1]$

(b) on $[-2,3]$

5.5 (cont)

Ex 4 Find abs. min/max pts (if they exist) for $f(x)=\frac{e^x}{x}$ on $(0,\infty)$.

5.6 Optimization (Story Problems :))

Ex 1 What quantity should be added to 5 + subtracted from 5 to produce the maximum product of the results?

Strategy

1. Write down info, assign variable name, draw picture (if applicable).
2. Find primary eqn (the one we need to maximize or minimize).
3. We may need to use a secondary eqn to get rid of a variable in primary eqn.
4. Take derivative of primary eqn, set it equal to 0 + solve.
5. make sure pt we get is really a max or min.

5.6 (cont)

Ex 2 Find the dimensions of a rectangle w/ area 225 ft^2 that has the least perimeter.
What is that perimeter?

5.6 (cont)

Ex 3 A company manufactures + sells x digital cameras per week. The weekly price-demand + cost equations are

$$p = 400 - 0.4x \quad + \quad C(x) = 2000 + 160x.$$

(a) What price should the company charge for the cameras + how many cameras should be produced to maximize revenue? What is max revenue?

(b) What is profit fn? Maximize profit.

5.6 (cont)

Ex 4 A 300-room hotel in Las Vegas is filled to capacity every night at \$80/room. For each \$1 increase in rent, 3 fewer rooms are rented. If each rented room costs \$10 to service per day, how much should they charge per room to maximize profit? what is max profit?