Math2210 Midterm 3 (12.9, 13.1-13.4, 13.6-13.9) Spring, 2015 Instructor: Kelly MacArthur

uid number: Key Special number: ________

Instructions:

- Please show all of your work as partial credit will be given where appropriate, **and** there may be no credit given for problems where there is no work shown.

- All answers should be completely simplified, unless otherwise stated.

- There are no calculators or any sort of electronics allowed on this exam. Make sure all cell phones are put away and out of sight. If you have a cell phone out at any point, for any reason, you will receive a zero on this exam.

- You will be given an opportunity to ask clarifying questions about the instructions at exactly 8:40 a.m. (for a couple minutes). The questions will be answered for the entire class. After that, no further questions will be allowed, for any reason.

- You must show us your U of U student ID card when finished with the exam.

- The exam key will be posted on Canvas by noon.

- You may ask for scratch paper. You may use NO other scratch paper. Please transfer all finished work onto the proper page in the test for us to grade there. We will not grade the work on the scratch page.

- You are allowed to use one 8.5x11 inch piece of paper with notes for your reference during the exam.

(This exam totals 70 points, not including the extra credit problem.)
STUDENT—PLEASE DO NOT WRITE BELOW THIS LINE. THIS TABLE IS TO BE USED FOR GRADING.

Problem	Score
1	
2	
3	
4	
5	
6	
EC (from Canvas)	

Raw total (out of 70): Total Percentage:	

1. True or False questions.

(a) (1 pt) $\int_a^b \int_c^d g(x,y)\,dx\,dy = \left(\int_a^b g(x,y)\,dx\right)\left(\int_c^d g(x,y)\,dy\right)$ T or F (circle one)

(b) (1 pt) $\int_a^b \int_c^d g(x)h(y)\,dy\,dx = \left(\int_a^b g(x)\,dx\right)\left(\int_c^d h(y)\,dy\right)$ T or F (circle one)

(c) (1 pt) $\{(\rho,\phi,\theta):\phi=\frac{\pi}{2}\} = \{(r,\theta,z):z=0\} = \{(x,y,z):z=0\}$ T or F (circle one)

these are all different ways to represent xy-plane

(d) (2 pts) $\int_0^2 \int_{-1}^1 \sin(x^3y^3)\,dx\,dy = 0$ T or F (circle one)

Explain your reasoning: $\sin(x^3y^3)$ is an odd function in x

so $\int_{-1}^1 \sin(x^3y^3)dx = 0$

(e) (2 pts) $\int_{-1}^1 \int_{-1}^1 e^{x^2+2y^2}\,dy\,dx = 4\int_0^1 \int_0^1 e^{x^2+2y^2}\,dy\,dx$ T or F (circle one)

$= \int_{-1}^1 \int_{-1}^1 e^{x^2}e^{2y^2}\,dy\,dx = \left(\int_{-1}^1 e^{x^2}dx\right)\left(\int_{-1}^1 e^{2y^2}dy\right) = \left(2\int_0^1 e^{x^2}dx\right)\left(2\int_0^1 e^{2y^2}dy\right)$

Explain your reasoning: $= 4\int_0^1 \int_0^1 e^{x^2}e^{2y^2}\,dx\,dy$ since e^{x^2} and e^{2y^2} are both even functions

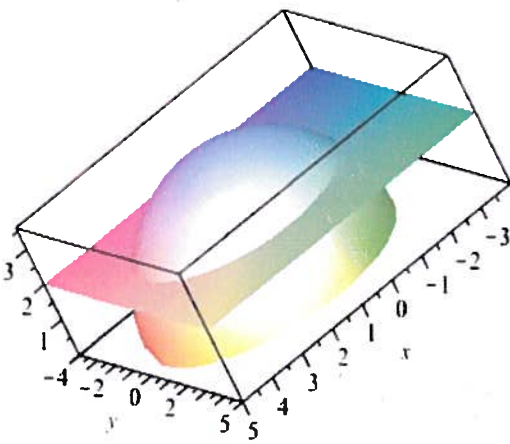
(f) (2 pts) $\int_{-1}^1 \int_{-1}^1 e^{x+2y}\,dy\,dx = 4\int_0^1 \int_0^1 e^{x+2y}\,dy\,dx$ T or F (circle one)

Explain your reasoning: because $e^{x+2y} = e^x e^{2y}$ and neither e^x nor e^{2y} are even functions

(g) (1 pt) $\int_a^b \int_a^b f(x)f(y)\,dx\,dy = \left(\int_a^b f(x)\,dx\right)^2$ T or F (circle one)

$= \left(\int_a^b F(x)dx\right)\left(\int_a^b f(y)dy\right) = \left(\int_a^b f(x)dx\right)^2$

2. (10 pts) Setup the integral to determine the volume of the solid bounded by the plane $z=2$ and the surface $(x-1)^2+y^2=13-z$. *(I want the solid that's above the plane and below the paraboloid.)* **(Do not evaluate.)** $\rightarrow z = 13-(x-1)^2-y^2$



$2 \le z \le 13-(x-1)^2-y^2$

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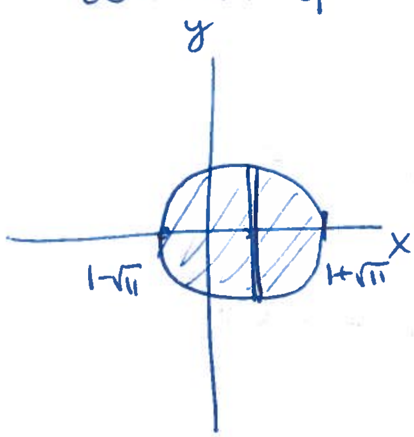


$2 = 13-(x-1)^2-y^2$

$(x-1)^2+y^2 = 11 \quad \Rightarrow y = \pm\sqrt{11-(x-1)^2}$

$-\sqrt{11-(x-1)^2} \le y \le \sqrt{11-(x-1)^2}$

$1-\sqrt{11} \le x \le 1+\sqrt{11}$

Volume Integral: $\displaystyle\int_{1-\sqrt{11}}^{1+\sqrt{11}} \int_{-\sqrt{11-(x-1)^2}}^{\sqrt{11-(x-1)^2}} \int_{2}^{13-(x-1)^2-y^2} dz\, dy\, dx$

(Do NOT evaluate/compute the integral. Just set it up.)

3. For the integral $V=\int_0^3\int_3^{12-y^2}\int_0^{\sqrt{12-z-y^2}} f(x,y,z)\;dx\,dz\,dy$, do the following.

$x=\sqrt{12-z-y^2}$

$\Rightarrow x^2+y^2=12-z$

or $z=12-(x^2+y^2)$

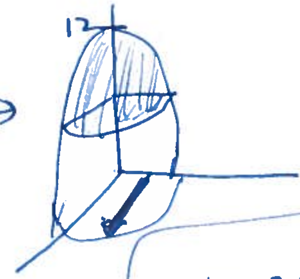
(a) (10 pts) Rewrite it, changing the order of integration to $dz\,dy\,dx$.
(Hint: Drawing the solid will be very useful here.)

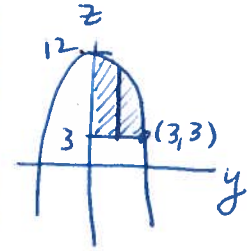
$0\le x\le\sqrt{12-z-y^2}$

$3\le z\le 12-y^2$

$0\le y\le 3$

here's the solid we want





when $z=3$!

$z=12-y^2 \Rightarrow 3=12-y^2$

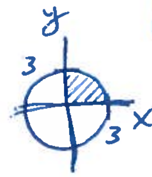
$y^2=9$

$y=\pm 3$

$3\le z\le 12-(x^2+y^2)$

$0\le y\le\sqrt{9-x^2}$

$0\le x\le 3$



$z=3$; $3=12-(x^2+y^2)$

$x^2+y^2=9$

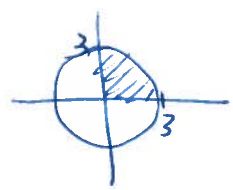
Answer (a): $\int_0^3\int_0^{\sqrt{9-x^2}}\int_3^{12-(x^2+y^2)} f(x,y,z)\,dz\,dy\,dx$

(b) (10 pts) Rewrite it using cylindrical coordinates.

$3\le z\le 12-r^2$

$0\le r\le 3$

$0\le\theta\le\pi/2$



$0\le r\le 3$

$0\le\theta\le\pi/2$

Answer (b): $\int_0^{\pi/2}\int_0^3\int_3^{12-r^2} f(r\cos\theta, r\sin\theta, z)\,r\,dz\,dr\,d\theta$

4. (10 pts) Use a triple integral in spherical coordinates to find the volume of the solid inside the sphere $x^2+y^2+z^2=25$ and outside the cone $z=\sqrt{3(x^2+y^2)}$ and above the xy-plane.

sphere: $\rho=5$

cone: $\varphi=\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)=\tan^{-1}\left(\frac{1/2}{\sqrt{3}/2}\right)$

$\Rightarrow \varphi=\frac{\pi}{6}$

$0\le\rho\le5$

$\frac{\pi}{6}\le\varphi\le\frac{\pi}{2}$

$0\le\theta\le2\pi$

$$V=\int_0^{2\pi}\int_{\pi/6}^{\pi/2}\int_0^5 \rho^2\sin\varphi\,d\rho\,d\varphi\,d\theta$$

$$=2\pi\left(\int_{\pi/6}^{\pi/2}\sin\varphi\,d\varphi\right)\left(\frac{\rho^3}{3}\Big|_0^5\right)$$

$$=2\pi\left(\frac{125}{3}\right)\left(-\cos\varphi\Big|_{\pi/6}^{\pi/2}\right)=\frac{250\pi}{3}\left(-\cos\frac{\pi}{2}-\left(-\cos\frac{\pi}{6}\right)\right)$$

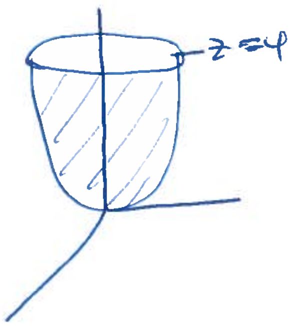
$$=\frac{250\pi}{3}\left(0+\frac{\sqrt{3}}{2}\right)=\frac{125\pi\sqrt{3}}{3}$$

Answer: $\frac{125\pi\sqrt{3}}{3}$

5. (10 pts) Find the <u>surface area</u> of the surface $z=x^2+y^2$ that is cut by the plane $z=4$.
(Note: Pay attention. Don't give me volume. Give me surface area!)

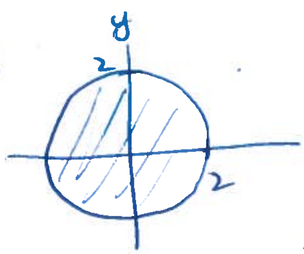
$f(x,y)=x^2+y^2$ $\qquad f_x=2x \qquad f_y=2y$

$\sqrt{f_x^2+f_y^2+1} = \sqrt{4x^2+4y^2+1}$



$$SA=\iint_S \sqrt{4x^2+4y^2+1}\,dy\,dx$$

S:



$x^2+y^2=4$

switch to polar coords

$0 \le r \le 2$
$0 \le \theta \le 2\pi$

$$\Rightarrow SA=\int_0^{2\pi}\int_0^2 \sqrt{4r^2+1}\; r\,dr\,d\theta$$

$u=4r^2+1$
$du=8r\,dr$
$\frac{1}{8}du=r\,dr$

$r=0,\ u=1$
$r=2,\ u=4(4)+1=17$

$$=\frac{2\pi}{8}\int_1^{17}\sqrt{u}\,du$$

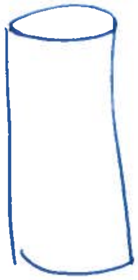
$$=\frac{\pi}{4}\left(\frac{2}{3}u^{3/2}\right)\Big|_1^{17}$$

$$=\frac{\pi}{6}\left(17\sqrt{17}-1\right)$$

Surface Area: $\frac{\pi}{6}\left(17\sqrt{17}-1\right)$

6. (10 pts) A right circular cylindrical can is to hold 4π cubic inches of frozen orange juice. The cost per square inch of constructing the metal top and bottom is 10 cents and the cost per square inch of constructing the cardboard side is 5 cents. What are the dimensions of the least expensive can? How do you know it's the minimum cost and not the maximum cost?

(Hint: This is the Lagrange multiplier problem.
Ask yourself: (1) What are you trying to minimize? Find a function for that. That's your f.
(2) What is the constraint that you must satisfy. Write an equation for that. That's your g.
You should get two functions in terms of r and h.)



$V = 4\pi \text{ in}^3$ (fixed)

$4\pi = \pi r^2 h \iff 4 = r^2 h$ (my "$g(r,h)$")

cost fn (in cents)

$C(r,h) = 10(2\pi r^2) + 5(2\pi r h)$

$= 20\pi r^2 + 10\pi r h$

my "$f(r,h)$"

we want to minimize C.

$\nabla C = \lambda \nabla g \implies \langle C_r, C_h \rangle = \lambda \langle g_r, g_h \rangle$

$\langle 40\pi r + 10\pi h, 10\pi r \rangle = \lambda \langle 2rh, r^2 \rangle$

① $40\pi r + 10\pi h = 2\lambda r h$ ② $10\pi r = \lambda r^2$ ③ $4 = r^2 h$

$10\pi = \lambda r$ (since $r > 0$)

$\lambda = \frac{10\pi}{r}$

$40\pi r + 10\pi h = 2\left(\frac{10\pi}{r}\right) r h$

$20\pi r + 5\pi h = 10\pi h$

$4r + h = 2h$

$4r = h$

$4 = r^2(4r)$

$1 = r^3$

$r = 1$

$\implies h = 4(1) = 4$

$\lambda = \frac{10\pi}{1} = 10\pi$

$\implies C(1,4) = 20\pi(1^2) + 10\pi(1)(4) = 60\pi$

How do I know it's min (not max)?

try: $h = 1$, then $4 = r^2(1) \implies r = 2$

$C(2,1) = 20\pi(2^2) + 10\pi(2)(1) = 100\pi$

$100\pi > 60\pi \implies$ we found min C-value.

Answer: dimensions: $r = 1$ in, $h = 4$ in