Math2210 Midterm 1 (10.4, 11.1-11.6, 11.8) Spring, 2015 Instructor: Kelly MacArthur

uid number: Key Special number: _________

Instructions:

- Please show all of your work as partial credit will be given where appropriate, **and** there may be no credit given for problems where there is no work shown.
- All answers should be completely simplified, unless otherwise stated.
- There are no calculators or any sort of electronics allowed on this exam. Make sure all cell phones are put away and out of sight. If you have a cell phone out at any point, for any reason, you will receive a zero on this exam.
- You will be given an opportunity to ask clarifying questions about the instructions at exactly 8:40 a.m. (for a couple minutes). The questions will be answered for the entire class. After that, no further questions will be allowed, for any reason.
- You must show us your U of U student ID card when finished with the exam.
- The exam key will be posted on Canvas by noon.
- You may ask for scratch paper. You may use NO other scratch paper. Please transfer all finished work onto the proper page in the test for us to grade there. We will not grade the work on the scratch page.
- You are allowed to use one 8.5x11 inch piece of paper with notes for your reference during the exam.

(This exam totals 100 points, not including the extra credit problem.)

STUDENT—PLEASE DO NOT WRITE BELOW THIS LINE. THIS TABLE IS TO BE USED FOR GRADING.

Problem	Score
1	
2 & 3	
4	
5	
6	
7 & EC	

Total Percentage:	

1. (5 points each) Let $a = \langle 2, 0, -5 \rangle$, $b = \langle 4, 1, 0 \rangle$ and $c = 3j - 2k$. Find each of the following.

(a) $a \cdot (b + c)$

$\vec{b} + \vec{c} = \langle 4+0, 1+3, 0+-2 \rangle = \langle 4, 4, -2 \rangle$

$\vec{a} \cdot (\vec{b} + \vec{c}) = \langle 2, 0, -5 \rangle \cdot \langle 4, 4, -2 \rangle$

$= 8 + 0 + 10 = 18$

$a \cdot (b + c)$ = 18

(b) projection of b onto a

$\text{pr}_{\vec{a}} \vec{b} = \left(\frac{\vec{a} \cdot \vec{b}}{\|\vec{a}\|} \right) \vec{a} = \frac{8+0+0}{2^2 + 5^2} \langle 2, 0, -5 \rangle$

$= \frac{8}{29} \langle 2, 0, -5 \rangle$

projection of b onto a = $\langle \frac{16}{29}, 0, \frac{-40}{29} \rangle$

(c) $a \cdot (b \times c)$

$$\vec{b} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 1 & 0 \\ 0 & 3 & -2 \end{vmatrix} = \hat{i}(-2-0) - \hat{j}(-8-0) + \hat{k}(12-0)$$

$= \langle -2, 8, 12 \rangle$

$\vec{a} \cdot (\vec{b} \times \vec{c}) = 2(-2) + 0(8) + -5(12) = -4 + 0 + -60 = -64$

$a \cdot (b \times c)$ = -64

2. (10 points) Find the equation of the sphere that has the line segment joining (1, 0, 5) and (3, 6, -1) as a diameter.

midpt = center = $\left(\frac{4}{2}, \frac{6}{2}, \frac{4}{2}\right) = (2,3,2)$

diameter

$= \sqrt{(1-3)^2+(0-6)^2+(5-1)^2}$

$= \sqrt{4+36+36}$

$= \sqrt{76} = 2\sqrt{19}$

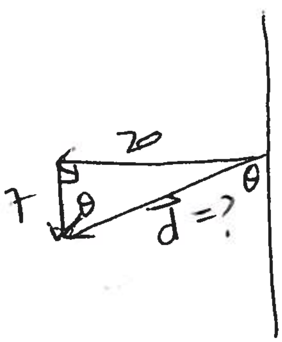
$\Rightarrow r = \frac{1}{2}(2\sqrt{19}) = \sqrt{19}$

Radius = $\sqrt{19}$ units

center = $(2,3,2)$

Eqn of sphere: $(x-2)^2+(y-3)^2+(z-2)^2=19$

3. (10 points) A luxury sail boat is traveling due west at 20 miles per hour. A woman on the ship is running across the ship, heading due south, at 7 miles per hour. What are the magnitude and direction of her velocity relative to the surface of the water? *(Since you don't have a calculator, just give the angle in simplified form.)*



$\|\vec{d}\| = \sqrt{7^2+20^2} = \sqrt{449}$

S $\arctan\left(\frac{20}{7}\right)$ W

$\tan\theta = \frac{20}{7}$

$\theta = \arctan\left(\frac{20}{7}\right)$

velocity magnitude: $\sqrt{449}$ mph

velocity direction (as an angle): S $\arctan\left(\frac{20}{7}\right)$ W

4. (20 points) For position vector given by $r(t)=5t\,\boldsymbol{i}-e^{t}\boldsymbol{j}+e^{2t}\boldsymbol{k}$, find the velocity and acceleration vectors, the speed at $t=\ln 2$ and parametric equations of the tangent line at $t=0$.

$v(t) = 5\hat{i} - e^{t}\hat{j} + 2e^{2t}\hat{k}$ or $\langle 5, -e^{t}, 2e^{2t} \rangle$

$a(t) = 0\hat{i} - e^{t}\hat{j} + 4e^{2t}\hat{k}$ or $\langle 0, -e^{t}, 4e^{2t} \rangle$

$\|\vec{v}(\ln 2)\| = \|\langle 5, -2, 2e^{\ln 4} \rangle\|$

$= \|\langle 5, -2, 8 \rangle\| = \sqrt{25+4+64} = \sqrt{93}$

speed at $t=\ln 2$ = $\sqrt{93}$

(answer should be simplified completely)

direct^n of line = $\vec{v}(0) = \langle 5, -1, 2 \rangle$

pt on line = $\vec{r}(0) = \langle 0, -1, 1 \rangle$

$x = 0 + 5t$

$y = -1 - t$

$z = 1 + 2t$

parametric equations of the tangent line when $t=0$

$x = 5t, \quad y = -1-t, \quad z = 1+2t$

5. (10 points each) For the planes given by $3x + y - z = 4$ ① and $2x - y + 4z = 5$ ②, answer the following questions.

(a) Find the line of intersection between the planes.

find a pt when $x=0$:

$$\begin{aligned} ①\ & y - z = 4 \\ +\ ②\ & -y + 4z = 5 \\ \hline & 3z = 9 \\ & z = 3 \end{aligned}$$

$\Rightarrow y - 3 = 4 \Rightarrow y = 7$

pt $(0, 7, 3)$ on line

directn of line:

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -1 \\ 2 & -1 & 4 \end{vmatrix}$$

$= \hat{i}(4-1) - \hat{j}(12+2) + \hat{k}(-3-2)$

$= \langle 3, -14, -5 \rangle$

Line: $x = 3t, \quad y = 7 + -14t, \quad z = 3 - 5t$

(b) Find the equation of the plane that is perpendicular to the line of intersection and goes through the point (-2, 1, 7).

$\vec{n} = \langle 3, -14, -5 \rangle \qquad$ pt $(-2, 1, 7)$

plane eqn:

$3x - 14y - 5z = D$

$3(-2) - 14(1) - 5(7) = D$

$-6 - 14 - 35 = D$

$D = -55$

Equation of plane: $3x - 14y - 5z = -55$

6. (15 points) For each surface, give its shape name, and draw a quick sketch of its graph. Possible surface shape names: *cylinder, plane, ellipsoid, sphere, hyperboloid of one sheet, hyperboloid of two sheets, elliptic paraboloid, hyperbolic paraboloid, elliptic cone.*

(a) $4x^2+9y^2+z^2-1=0$ type of surface: ellipsoid

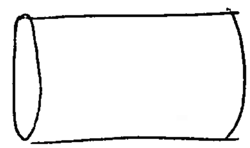
$\frac{x^2}{1/4}+\frac{y^2}{4/9}+\frac{z^2}{1}=1$

Quick sketch:

(b) $4x^2+z^2-4=0$ type of surface: cylinder

$\frac{x^2}{1}+\frac{z^2}{4}=1$

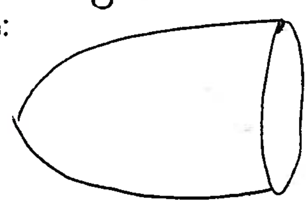
Quick sketch:



(c) $4x^2-9y+z^2=0$ type of surface: elliptic paraboloid

$9y=4x^2+z^2$ $\quad y=\frac{x^2}{9/4}+\frac{z^2}{9}$

Quick sketch:



(d) $4x^2-9y^2+z^2+4=0$ type of surface: hyperboloid in 2 sheets

$x^2-\frac{y^2}{4/9}+\frac{z^2}{4}=-1 \iff -x^2+\frac{y^2}{4/9}-z^2/4=1$

Quick sketch:

(e) $4x^2-9y^2+z^2-4=0$ type of surface: hyperboloid in 1 sheet

$x^2-\frac{y^2}{4/9}+\frac{z^2}{4}=1$

Quick sketch:

7. (10 points) Find the arc length of the curve given by the position vector
$r(t)=8i+\sin(e^t)j-\cos(e^t)k$ for $0 \le t \le \ln(5)$

$$L=\int_0^{\ln 5}\sqrt{\|\vec{r}\,'(t)\|^2}\,dt = \int_0^{\ln 5}\|\vec{r}(t)\|\,dt$$

$$=\int_0^{\ln 5}\sqrt{0+(e^t\cos(e^t))^2+(e^t\sin(e^t))^2}\,dt$$

$$=\int_0^{\ln 5}\sqrt{e^{2t}(\cos^2(e^t)+\sin^2(e^t))}\,dt$$

$$=\int_0^{\ln 5}\sqrt{e^{2t}}\,dt=\int_0^{\ln 5}e^t\,dt=e^t\Big|_0^{\ln 5}$$

$$=e^{\ln 5}-e^0=5-1=4$$

arc length: 4

Extra Credit: (5 points) Your class grade is based on six components: overall daily quiz score, overall weekly quiz score, three midterms and a final exam, weighted 10%, 15%, 20%, 20%, 10% and 25% respectively. Assume your grades on these components are given by d, w, m_1, m_2, m_3, f, respectively.

What do the vectors $w=\langle 0.1, 0.15, 0.2, 0.2, 0.1, 0.25\rangle$ and $y=\langle d, w, m_1, m_2, m_3, f\rangle$ represent, in terms of the course?

$\vec{w}$ is the vector of weights
$\vec{y}$ is the grade vector, where each component represents one section of the grade

Calculate the dot product $w \cdot y$.

$$\vec{w}\cdot\vec{y}=0.1d+0.15w+0.2m_1+0.2m_2+0.1m_3+0.25f$$

What does that dot product represent, in terms of the course?

the overall grade for the class as its weighted average of the components