let \( a_n \rightarrow 0 \) and suppose that \( \sum a_n \) converges. Prove that \( \sum a_n^2 \) also converges.

**PF** since \( \sum a_n \) converges, then we know 
\[
\lim_{n \to \infty} a_n = 0
\]
by \( n^{th} \) term test for divergence.

(Which states clearly that \( \sum a_n \) converges 
\( \Rightarrow \) \( \lim_{n \to \infty} a_n = 0 \).

Then there exists an \( N \) large enough so that \( \forall n > N \), \( a_n^2 < a_n \) (since \( \forall x \in (0,1) \) \( x^2 < x \)) because for these \( n \) values, \( a_n < 1 \).

By OCT, since \( \sum a_n \) converges and \( \sum a_n^2 < \sum a_n \), then so does \( \sum a_n^2 \) converge.

\[ \checkmark \]

**9.5 #31** Claim: \( \sum a_n \) diverges \( \Rightarrow \) \( \sum |a_n| \) diverges.

**PF** cleverest way to do this is to invoke the Absolute Convergence Test (pg 476). The contrapositive statement for that test is:

If \( \sum a_n \) doesn’t converge, then \( \sum |a_n| \) doesn’t converge, which is exactly our claim. \( \checkmark \)
9.5 #33 1. Show that positive terms of alternating harmonic series form a diverged series.
2. Show the same for the series of negative terms.

Pf

alt. harmonic series:

\[ \sum_{n=1}^{\infty} \frac{(-1)^n}{n} = -1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \ldots \]

1. \[ \frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8} + \ldots = \frac{\infty}{\infty} = \frac{1}{2} \frac{\infty}{n=1} \quad \text{(p-series, p=1)} \]
   diverges

2. \[ -\frac{1}{3} - \frac{1}{5} - \frac{1}{7} - \ldots = -\frac{\infty}{\infty} = \frac{1}{2} \frac{1}{2n-1} \quad \text{by Let, } \omega \frac{1}{n} \frac{1}{n} \quad \text{so } 2b_n \text{ diverges} \]
   we get \( \lim_{n \to \infty} \frac{N}{2n-1} = \frac{1}{2} < \infty \)
   \( \Rightarrow \frac{\infty}{n=1} \frac{1}{2n-1} \text{ also diverges} \)

\[ \Rightarrow -\frac{\infty}{N=1} \frac{1}{2n-1} \text{ diverges} \]