5.2/5.3 Practice (Volumes of Solids of Revolution)

Ex 1 Find the volume of the solid generated by the indicated region being revolved about the given axis.

Disk method

$V = \pi \int_a^b r^2 \, dx$ (or dy)

r = radius of disk

Washer method

$V = \pi \int_a^b (r_o^2 - r_i^2) \, dx$ (or dy)

(r_o = r-outer ; r_i = r-inner)

Shell method

$V = 2\pi \int_a^b rh \, dx$ (or dy)

r = radius of shell
h = height of shell

(a) $y = x^{2/3}$, $y = 0$, $x = -2$, $x = 3$
about the x-axis

(b) $y = x^{2/3}$, $y = 0$, $x = -2$, $x = 3$ about the line
$y = -1$

Ex 2 Set up the volume integrals.

$x^2+y^2=4$, $y=0$, $x=0$, $x=1$ (a) about x-axis

(b) about y-axis

(c) about $x=2$

<u>Ex 3</u> Setup these volume integrals.

$y = -2x^2 + 4x + 3, \; y = 2.$

(a) about y-axis

(b) about x-axis

(c) about $x = -1$.

<u>Ex 4</u> (#19) A round hole of radius a is drilled through the center of a solid sphere of radius b ($b>a$). Find the volume of the remaining solid.

5.4 Practice (Length of a Curve / Surface Area)

Ex 1 Find the length of the indicated curve.

(a) $30xy^3 - y^8 = 15$ between $y=1$ and $y=3$

L = arc length

$L = \int_a^b ds$

① $ds = \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$

or

② $ds = \sqrt{1+\left(\frac{dx}{dy}\right)^2}\, dy$

or

③ $ds = \sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\, dt$

(b) $x = a\cos t + at\sin t$, $y = a\sin t - at\cos t$, $t \in [-1, 1]$ (a is a constant)

Ex 2 Find surface area.

revolve $y = \frac{x^6+2}{8x^2}$ $x \in [1,3]$

about x-axis

Surface Area

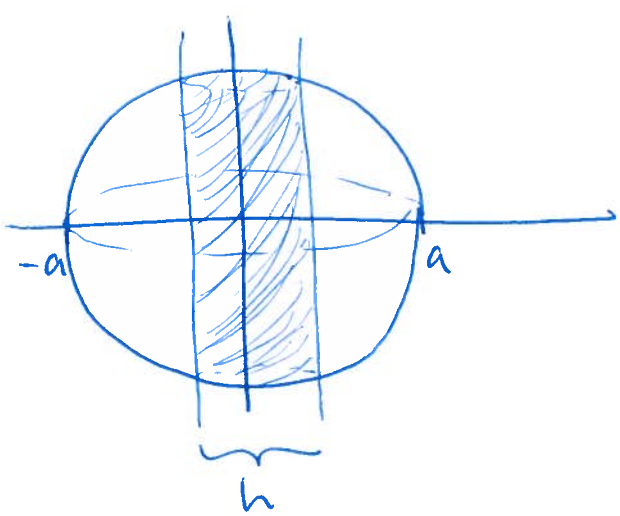
$SA = \int_a^b 2\pi f(x)\, ds$

$ds = \sqrt{1+(f'(x))^2}\, dx$

only for $y=f(x)$

and rotated about x-axis

Ex 3 Show that the area of the part of the surface of a sphere of radius a between two parallel planes h units apart $(h < 2a)$ is $2\pi a h$.



5.5 Practice (Work)

$W = \int_a^b F(x)\,dx$

$F(x)$ = force

Ex 1 For a certain type of nonlinear spring, the force required to keep the spring stretched a distance s is given by $F = ks^{4/3}$. If the force required to keep it stretched 8 inches is 2 pounds, how much work is done in stretching this spring 27 inches?

Ex2 A 10-pound monkey hangs at the end of a 20-foot chain that weighs $\frac{1}{2}$ pound/foot. How much work does it do in climbing the chain to the top? (Assume the end of the chain is attached to the monkey.)

5.6 Practice (Moments and Center of Mass)

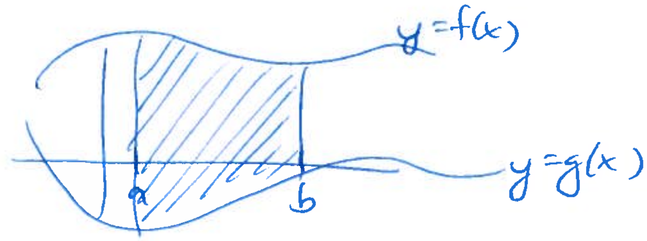
Ex 1 Find the centroid of the region bounded by the given curves.

(a) $y = x^2$, $y = x + 3$

mass $m = \delta \int_a^b (f(x) - g(x))\,dx$

$M_y = \delta \int_a^b x\,(f(x) - g(x))\,dx$

$M_x = \frac{\delta}{2} \int_a^b [f^2(x) - g^2(x)]\,dx$



$\bar{x} = \frac{M_y}{m}$, $\bar{y} = \frac{M_x}{m}$

$(\bar{x}, \bar{y})$ = center of mass (or centroid)

for homogeneous lamina

(b) $x = y^2 - 3y - 4$, $x = -y$