4.1 Practice (Intro to Area)

Ex. 1 Write in sigma notation.

(a) \( a_1 - a_2 + a_3 - a_4 + a_5 \)

(b) \( \frac{1}{3} + \frac{2}{4} + \frac{3}{5} + \frac{4}{6} + \frac{5}{7} \)

Ex. 2 Find each sum (use special sum formulas).

(a) \( \sum_{k=1}^{20} (2k^2 - 3) \)

\( \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \)

\( \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \)
(b) \[ \sum_{j=1}^{n} (3j+1)^2 \]

4.2 Practice (Definite Integral)

Ex. 1 Calculate, using defn of definite integral.

(a) \[ \int_{-1}^{2} (3x^2 + 5) \, dx \]

\[
\text{Definite Integral} = \left. \sum_{i=1}^{n} f(x_i) \, \Delta x \right|_{a}^{b} = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i) \, \Delta x
\]

\[ \Delta x = \frac{b-a}{n} \]

\[ x_i = a + i \Delta x \]

(right end pt)
Ex 2: \[ f \text{ is odd fn, } \; \text{g is even fn; } \int_{-1}^{1} f(x) \, dx = \int_{-1}^{1} g(x) \, dx = 3 \]

Use geometric thinking to evaluate:

(a) \[ \int_{-1}^{1} f(x) \, dx \]
(b) \[ \int_{-1}^{1} g(x) \, dx \]
(c) \[ \int_{-1}^{1} |f(x)| \, dx \]
(d) \[ \int_{-1}^{1} f(x) g(x) \, dx \]
4.3 Practice (First Fundamental Theorem of Calculus)

Ex1 Find formula for accumulation for \( A(x) \) to represent area.

\[
\frac{d}{dx} \left[ \int_a^x f(t) \, dt \right] = f(x)
\]

\( a = \text{constant} \)

\( f(x) \) continuous on \( [c, d] \)

\( a, x \in [c, d] \)

Ex2 Find \( G(x) \).

(a) \( G(x) = \int_x^1 \sqrt{t^2 + 1} \, dt \)

(b) \( G(x) = \int_2^{\tan x} e^{-t^2} \, dt \)
Ex. 3 Find \( G'(x) \).

\[ G(x) = \int_x^4 \sec(t) \, dt \]

4.4 Practice (Second Fundamental Theorem of Calculus)

Ex. 1 Evaluate.

(a) \[ \int_1^3 \frac{x^4 - 5}{x^2} \, dx \]
(b) \[ \int_{0}^{1} (x^{4/3} - 2x^{4/3}) \, dx \]

Ex. 2. Use u-substitution to evaluate.

(a) \[ \int x^3 \cos(x^4 + 1) \, dx \]
(b) \[ \int x^3 \sec(x^2 - 3) \tan(x^2 - 3) \sqrt{\sec(x^2 - 3)} \, dx \]

EX 3

Evaluate.

(a) \[ \int_{1}^{2} \frac{x^3 + 2}{\sqrt{x^4 + 8x}} \, dx \]
(b) \[ \int_{1}^{4} \frac{(\sqrt[3]{x} - 1)^3}{\sqrt{x}} \, dx \]
4.5 Practice (Mean Value Theorem (MVT) for Integrals)

**Example 1** Find the average value on \([0, \pi/2]\) of \(f(x) = \sin^2 x \cos x\).

\[ f(c) = \frac{1}{b-a} \int_a^b f(x) \, dx \quad \text{called \textit{avg. value}} \]

\( f(x) \) continuous on \([a, b]\)

\( c \in (a, b) \)

**Example 2** Find all values \(a \leq c \leq b\) guaranteed by MVT, on \([0, 2]\) for \(f(x) = x^3\).
Ex 3 Use symmetry to help evaluate.

(a) \[ \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} (x\sin^3 x + x^2 + \tan x) \, dx \]

(b) \[ \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} (\sin x - \cos x)^2 \, dx \]
5.1 Practice (Area of a Plane Region)

Ex. 1 Find area between these curves.

(a) \( y = \sqrt{x} \), \( y = x - 4 \), \( x = 0 \)

(b) \( x = (3-y)(y+1) \), \( x = 0 \)
Ex2  Find area between curves:

(a) \( y = (x-3)(x-1) \), \( y = x \).

(b) \( x = 4y^4 \), \( x = 8 - 4y^4 \).