How to Interpret Results of Tests for Infinite Series $(\sum a_n)$ Divergence / Convergence

① n^{th} term test

condition: none

test: $\lim_{n\to\infty} |a_n| = L$

result: if $L \neq 0$, $\sum a_n$ diverges

② (a) Geometric Series

condition: none

test: of form $\sum_{k=n}^{\infty} ar^k$

result if $|r|<1$, then $\sum_{k=n}^{\infty} ar^k$ converges (absolutely) to

$$\frac{\text{first term}}{1-r}$$

if $|r| \geq 1$, series diverges

(b) p-series

condition: $n \geq 1$ (starting value)

test: of form $\sum_{k=n}^{\infty} \frac{1}{k^p}$

result: if $p>1$, series converges (absolutely)

if $p \leq 1$, series diverges

③ (a) LCT

condition: none

test: choose b_n (usually a p-series)

$\lim_{n\to\infty} \left|\frac{a_n}{b_n}\right| = L$

result if $L<\infty$ 1) and $\sum b_n$ converges (absolutely)

then $\sum a_n$ converges

2) and $\sum b_n$ diverges, then $\sum a_n$ diverges (★)

or if $L=0$ and $\sum b_n$ converges (absolutely),

then $\sum a_n$ also converges

(b) ART

condition: none

test: $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \rho$

result: if $\rho<1$, series converges absolutely

if $\rho>1$, series diverges (★)

if $\rho=1$, no information

(★) if original series is not always positive, then we still have to check for conditional convergence

④ (a) IT (Integral Test)

$f(x)$ is continuous version of a_n

condition: ① continuous f
② all positive terms
③ nonincreasing

test: For $\sum_{k=n}^{\infty} a_k$,

compute $\int_n^{\infty} f(x)dx = L$

result: if $L<\infty$, series converges (absolutely)

if $\int_n^{\infty} f(x)dx$ diverges, so does series

(b) OCT (Ordinary Comp. Test)

condition: all positive terms

test: choose b_n that we know about $\sum b_n$ result

result: (1) if $a_n \le b_n$, then if $\sum b_n$ converges, so does $\sum a_n$ (absolutely)

(2) if $a_n \ge b_n$, and $\sum b_n$ diverges, then so does $\sum a_n$

⑤ Partial Sums argument

condition: none (* but this is mostly used for collapsing sums)

test: Find formula for S_n (n^{th} partial sum)

then take $\lim_{n\to\infty} S_n = L$.

result if $L<\infty$, series converges

if $L = \pm\infty$, series diverges

⑥ AST (Alternating Series Test)

condition: it must be an alternating series

test $\lim_{n\to\infty} a_n = L$

result if $L=0$, series converges conditionally

if $L \ne 0$, series diverges