(If Σaₙ converges, it converges absolutely.)

Yes: Is the series Σaₙ positive?

(If Σaₙ converges, it converges absolutely.)

Yes: Consider Σ|aₙ|. Σ|aₙ| is positive. Does Σ|aₙ| converge? (If Σ|aₙ| converges, converges absolutely)

No: Find lim₀ₙ⁻→∞ aₙ. Is lim₀ₙ⁻→∞ aₙ = 0?

No: diverges

Yes: diverges

p > 1 converges

p ≤ 1 diverges

Is it geometric? 

Yes: Σrⁿ

1 < r ≤ 1 converges to 1/(1-r)

1 > 1 diverges

No: Try Ratio Test:

Let bₙ = expression of only highest terms.

Calculate ρ = \lim₀ₙ⁻→∞ \frac{aₙ₊₁}{aₙ}

ρ > 1 converges

ρ = 1 or ρ doesn’t exist

ρ ≤ 1 diverges

Does aₙ involve only constant powers of n?

Yes: Let bₙ = substitute into Σaₙ

\cos(\frac{π}{n}) \approx 1 - \frac{1}{2} \frac{π²}{n²}

\sin(\frac{π}{n}) \approx \frac{1}{n}

\ln(1 + \frac{1}{n}) \approx \frac{1}{n}

No: Try a different bₙ and repeat or continue on.

Does aₙ involve sin(n) or cos(n)?

Yes: Use -1 ≤ sin, cos ≤ 1 to compare aₙ to a bₙ without sin(n) or cos(n).

Use Ordinary Comparison Test:

\[ aₙ ≤ bₙ \] converges

\[ bₙ ≤ aₙ \] diverges

Try a different bₙ or continue.

Does aₙ involve something you can integrate?

Yes: Try Integral Test:

Evaluate \[ \int_{0}^{∞} f(x)dx \] where \( f(n) = aₙ \)

f(x) continuous, positive, non-increasing

\[ ∫_{0}^{∞} f(x)dx \] and Σaₙ do the same thing.

No: diverges

Yes: does Σbₙ converge? diverges

Does Σbₙ converge? diverges

You should have discovered the answer by now. You may have to use the Comparison Test in conjunction with the Integral Test when aₙ involves transcendental functions. Otherwise you may have to resort to looking at partial sums.