Notes on Limits

Setup: You have to find $\lim_{x \to c} f(x)$

1. If $c$ is a finite number, first try plugging in $c$.
   - If you get a finite number back, then you're done.
   - If you get the $\frac{0}{0}$ case, then you need to simplify more to find the limit. Keep going until you can plug in $c$ for $x$.
   - If you get the $\frac{\text{finite number}}{0}$ case, then it will either (1) go to $\infty$, (2) go to $-\infty$, or (3) it will not exist (DNE). You need to check the right and left-hand limits.
     - If the RH limit and the LH limit both go to $\infty$, then the limit also goes to $\infty$.
     - If the RH limit and the LH limit both go to $-\infty$, then the limit also goes to $-\infty$.
     - If the RH limit goes to $-\infty$ and the LH limit goes to $\infty$ (or the other way around), then the limit does not exist (DNE).

2. If $c$ is $\pm \infty$ (and $f(x)$ is a rational function or at least has numerator and denominator that can be written as a collection of terms to powers), then
   - If the highest degree of the numerator > highest degree of the denominator, then the limit goes to $\pm \infty$. (You have to analyze the particular problem to decide if it's positive or negative infinity.)
   - If the highest degree of the numerator = highest degree of the denominator, then the limit is the quotient of the leading coefficients.
   - If the high degree of the numerator < highest degree of the denominator, then the limit is zero.
Asymptotes:

1. **To find Vertical Asymptotes (VA)===>**
   Look for x-values that will make the function undefined (e.g. x-values that make the denominator zero). Let's say that x = b makes function undefined.

   - If \( \lim_{x \to b} f(x) \) equals a finite number, then there is only a “hole” (a.k.a. removable discontinuity) at x = b.
   - If \( \lim_{x \to b} f(x) \) does not exist (or goes to \( \pm\infty \)), then x = b is a VA.

   *(A quick way to determine this is to try plugging in x = b in the function. If it goes to \( \frac{0}{0} \), then it’s a “hole.” If it goes to \( \frac{\text{finite number}}{0} \), then it’s a VA.)*

   A graph NEVER touches or crosses a vertical asymptote!!

2. **To find Horizontal Asymptotes (HA)===>**
   The HA can be found by calculating \( y = \lim_{x \to \pm\infty} f(x) \). If the limit goes to \( \pm\infty \), then there is no HA.

   *(If there is no HA and you have a rational function, you can always find the “slant asymptote” by doing long division.)*

   Remember that the HA just describes the behavior of the graph as x gets really huge (either negatively or positively). A graph can cross and touch the horizontal/slant asymptotes as many times as it wants...but as x gets huge, it will only approach the asymptote.