22 If the arc described in Exercise 21 is revolved about the x-axis, find the area of the resulting surface.

23 Prove that the surface area of a right circular cone of altitude a and base radius b is $\pi b\sqrt{a^2 + b^2}$.

24 If the ellipse $b^2x^2 + a^2y^2 = a^2b^2$ is revolved about the y-axis, find the area of the resulting surface.

25 A **torus** is the surface generated by revolving a circle about a nonintersecting line in its plane. Find the surface area of the torus generated by revolving the circle $x^2 + y^2 = a^2$ about the line $x = b$, where $0 < a < b$.

26 The shape of a reflector in a searchlight is obtained by revolving a parabola about its axis. If the reflector is 4 feet across at the opening and 1 foot deep, find its surface area.

13.8 REVIEW

Concepts

Define or discuss each of the following.

1 Plane curves

2 Closed curves

3 Parametric equations of a curve

4 Polar coordinate systems

5 Graphs of polar equations

6 Areas in polar coordinates

7 Length of a curve

8 Surfaces of revolution

Exercises

In Exercises 1–3 sketch the graph of the curve, find a rectangular equation of a graph which contains the points on the curve, and find the slope of the tangent line at the point corresponding to $t = 1$.

1 $x = 1/t + 1,\ y = 2/t - t;\ 0 < t \le 4$

2 $x = \cos^2 t - 2,\ y = \sin t + 1;\ 0 \le t \le 2\pi$

3 $x = \sqrt{t},\ y = e^{-t};\ t \ge 0$

4 Let C be the curve with parametric equations $x = t^2, y = 2t^3 + 4t - 1$, where t is in $\mathbb{R}$. Find the abscissas of the points on C at which the tangent line passes through the origin.

5 Compare the graphs of the following curves, where t is in $\mathbb{R}$.
(a) $x = t^2,\ y = t^3$
(b) $x = t^4,\ y = t^6$
(c) $x = e^{2t},\ y = e^{3t}$
(d) $x = 1 - \sin^2 t,\ y = \cos^3 t$

In each of Exercises 6–19, sketch the graph of the equation and find an equation in x and y which has the same graph.

6 $r = 3 + 2\cos\theta$

7 $r = 6\cos 2\theta$

8 $r = 4 - 4\cos\theta$

9 $r^2 = -4\sin 2\theta$

10 $r = 2\sin 3\theta$

11 $r(3\cos\theta - 2\sin\theta) = 6$

12 $r = e^{-\theta},\ \theta \geq 0$

13 $r^2 = \sec 2\theta$

14 $r = 8\sec\theta$

15 $r = 4\cos^2(\theta/2)$

16 $r = 6 - r\cos\theta$

17 $r - 1 = 0$

18 $r = \dfrac{8}{1 - 3\sin\theta}$

19 $r = \dfrac{8}{3 + \cos\theta}$

Find polar equations for the graphs in Exercises 20 and 21.

20 $x^2 + y^2 = 2xy$

21 $y^2 = x^2 - 2x$

22 Find a polar equation of the hyperbola which has focus at the pole, eccentricity 2, and equation of directrix $r = 6\sec\theta$.

23 Find the area of the region bounded by one loop of the graph of $r^2 = 4\sin 2\theta$.

24 Find the area of the region which is inside the graph of $r = 3 + 2\sin\theta$ and outside the graph of $r = 4$.

25 The position (x, y) of a particle at time t is given by $x = 2\sin t$, $y = \sin^2 t$. Find the distance the particle travels from $t = 0$ to $t = \pi/2$.

26 Find the length of the spiral $r = 1/\theta$ from $(1, 1)$ to $(1/2, 2)$.

27 Find the area of the surface generated by revolving the graph of $y = \cosh x$ from $x = 0$ to $x = 1$ about the x-axis.

28 The curve $x = 2t^2 + 1$, $y = 4t - 3$ where $0 \leq t \leq 1$ is revolved about the y-axis. Find the area of the resulting surface.

29 The arc of the spiral $r = e^{\theta}$ from $(1, 0)$ to $(e, 1)$ is revolved about the line $\theta = \pi/2$. Find the area of the resulting surface.

30 Find the area of the surface generated by revolving the lemniscate $r^2 = a^2\cos 2\theta$ about the polar axis.

9 Unit vector

10 Direction angles and direction cosines

11 The dot product and its properties

12 The vector product and its properties

13 The angle between two vectors

14 Parallel vectors

15 Orthogonal vectors

16 Cauchy-Schwarz Inequality

17 Triangle inequality

18 Work done by a force

19 Equations of a line

20 Equations of a plane

21 Cylinders

22 Quadric surfaces

23 Spherical coordinates

24 Cylindrical coordinates

Exercises

Find the vectors or scalars indicated in Exercises 1–10 if $\mathbf{a} = 2\mathbf{i} + 5\mathbf{j}$ and $\mathbf{b} = 4\mathbf{i} - \mathbf{j}$.

1 $4\mathbf{a} + \mathbf{b}$

2 $2\mathbf{a} - 3\mathbf{b}$

3 $(\mathbf{a} + \mathbf{b}) \cdot (\mathbf{a} - 2\mathbf{b})$

4 $3\mathbf{a} \cdot (5\mathbf{b} + \mathbf{i})$

5 $|\mathbf{a}| - |\mathbf{b}|$

6 $|\mathbf{a} - \mathbf{b}|$

7 The angle between **a** and **i**

8 A unit vector having the same direction as **b**

9 A unit vector orthogonal to **a**

10 The cosine of the angle between **a** and **b**

11 Given the points $A(5, -3, 2)$ and $B(-1, -4, 3)$, find the following.
(a) $d(A, B)$
(b) The coordinates of the midpoint of the line segment AB
(c) An equation of the sphere with center B and tangent to the xz-plane
(d) An equation of the plane through B parallel to the xz-plane
(e) Parametric equations for the line through A and B
(f) An equation of the plane through A with normal vector $\overrightarrow{AB}$

12 Find an equation for the plane through $A(0, 4, 9)$ and $B(0, -3, 7)$ which is perpendicular to the yz-plane.

13 Find an equation for the plane with x-intercept 5, y-intercept -2, and z-intercept 6.

14 Find an equation for the cylinder which is perpendicular to the xy-plane and has, for its directrix, the circle in the xy-plane with center $C(4, -3, 0)$ and radius 5.

15 Find an equation for an ellipsoid with center O which has x-intercept 8, y-intercept 3, and z-intercept 1.

16 Find an equation for the surface obtained by revolving the graph of the equation $z = x$ about the z-axis.

In each of Exercises 17–27, sketch the graph of the given equation.

17 $x^2 + y^2 + z^2 - 14x + 6y - 8z + 10 = 0$

18 $4y - 3z - 15 = 0$

19 $3x - 5y + 2z = 10$

20 $y = z^2 + 1$

21 $9x^2 + 4z^2 = 36$

22 $x^2 + 4y + 9z^2 = 0$

23 $z^2 - 4x^2 = 9 - 4y^2$

24 $2x^2 + 4z^2 - y^2 = 0$

25 $z^2 - 4x^2 - y^2 = 4$

26 $x^2 + 2y^2 + 4z^2 = 16$

27 $x^2 - 4y^2 = 4z$

If $\mathbf{a} = 3\mathbf{i} - \mathbf{j} - 4\mathbf{k}$, $\mathbf{b} = 2\mathbf{i} + 5\mathbf{j} - 2\mathbf{k}$, and $\mathbf{c} = -\mathbf{i} + 6\mathbf{k}$, find the vectors or scalars in Exercises 28–44.

28 $3\mathbf{a} - 2\mathbf{b}$

29 $\mathbf{a} \cdot (\mathbf{b} - \mathbf{c})$

30 $|\mathbf{b} + \mathbf{c}|$

31 $|\mathbf{b}| + |\mathbf{c}|$

32 A unit vector having the same direction as **a**

33 The direction cosines of **a**

34 The cosine of the angle between **a** and **c**

35 $\mathbf{a} \times \mathbf{b}$

36 $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$

37 $\text{comp}_{\mathbf{b}}\, \mathbf{a}$

38 $\text{comp}_{\mathbf{a}}\, (\mathbf{b} \times \mathbf{c})$

39 $\mathbf{a} \cdot \mathbf{a}$

40 $\mathbf{a} \times \mathbf{a}$

41 $(\mathbf{a} \times \mathbf{c}) + (\mathbf{c} \times \mathbf{a})$

42 $\mathbf{a} \times (\mathbf{b} + \mathbf{c})$

43 A vector having the opposite direction of **b** and twice the magnitude of **b**

44 Two unit vectors orthogonal to both **b** and **c**

45 Given the points $P(2, -1, 1)$, $Q(-3, 2, 0)$, and $R(4, -5, 3)$ find the following.
(a) The direction cosines of $\overrightarrow{PR}$
(b) A unit normal vector orthogonal to the plane determined by P, Q, and R
(c) An equation for the plane determined by P, Q, and R
(d) Parametric equations for a line through P which is parallel to the line through Q and R
(e) $\overrightarrow{QP} \cdot \overrightarrow{QR}$
(f) The angle between $\overrightarrow{QP}$ and $\overrightarrow{QR}$
(g) The area of the triangle with vertices P, Q, and R

46 Find the angle between the two lines $(x - 3)/2 = (y + 1)/(-4) = (z - 5)/8$ and $(x + 1)/7 = (6 - y)/2 = (2z + 7)/(-4)$.

47 Find parametric equations for each of the lines in Exercise 46.

48 Determine whether the following lines intersect and, if so, find the point of intersection:

$$x = 2 + t, y = 1 + t, z = 4 + 7t;\ x = -4 + 5t, y = 2 - 2t, z = 1 - 4t.$$

49 Find the angle between the lines in Exercise 48.

50 The position of a particle at time t is $(t, t\sin t, t\cos t)$. Find the distance the particle moves during the time interval $[0, 5]$.

51 If the rectangular coordinates of a point P are $(2, -2, 1)$, find cylindrical and spherical coordinates for P.

52 If spherical coordinates of a point P are $(12, 3\pi/4, \pi/6)$, find cylindrical and rectangular coordinates for P.

Find a rectangular equation and describe the graph of each equation in Exercises 53–56.

53 $\phi = 3\pi/4$

54 $r = \cos 2\theta$

55 $\rho \sin\phi \cos\theta = 1$

56 $\rho^2 - 3\rho = 0$

Find equations in cylindrical coordinates and in spherical coordinates for the graphs of the equations in each of Exercises 57–60.

57 $x^2 + y^2 = 1$

58 $z = x^2 - y^2$

59 $x^2 + y^2 + z^2 - 2z = 0$

60 $2x + y - 3z = 4$

15.7 REVIEW

Concepts

Define or discuss each of the following.

1 Vector-valued function

2 Limit of a vector-valued function

3 Continuity of a vector-valued function

4 Derivative of a vector-valued function

5 Tangent vector to a curve

6 Velocity and acceleration

7 Differentiation rules for vector-valued functions

8 Integrals of vector-valued functions

9 Curvature of a plane curve

10 Curvature of a space curve

11 Circle of curvature

12 Radius of curvature

13 Tangential component of acceleration

14 Normal component of acceleration

15 Kepler's Laws

Exercises

1 If $\mathbf{r}(t) = t^2\mathbf{i} + (4t^2 - t^4)\mathbf{j}$,
(a) sketch the curve determined by the components of $\mathbf{r}(t)$.
(b) find $\mathbf{r}'(t)$ and $\mathbf{r}''(t)$.
(c) sketch geometric vectors corresponding to $\mathbf{r}'(t)$ and $\mathbf{r}''(t)$ if $t = 0$, $t = 1$, and $t = 2$.

2 The position of a particle moving in a plane is given by

$$\mathbf{r}(t) = (t - \sin t)\mathbf{i} + (1 - \cos t)\mathbf{j}.$$

Find its velocity, acceleration, and speed at time t. Sketch the path of the particle together with geometric vectors corresponding to the velocity and acceleration for the following values of t.

(a) $t = 0$	(b) $t = \pi/4$	(c) $t = \pi/2$
(d) $t = 3\pi/4$	(e) $t = \pi$	(f) $t = 5\pi/4$
(g) $t = 3\pi/2$	(h) $t = 7\pi/4$	(i) $t = 2\pi$

3 If the curve C is given by $x = e^t \sin t, y = e^t \cos t, z = e^t$, where $0 \le t \le 1$, find
(a) a unit tangent vector to C at the point P corresponding to $t = 0$.
(b) the length of C.

4 The position of a particle at time t is given by

$$\mathbf{r}(t) = 3t\mathbf{i} + t^3\mathbf{j} + t^4\mathbf{k}.$$

(a) Find the velocity and acceleration at time t.
(b) Sketch the path of the particle together with vectors corresponding to the velocity and acceleration at $t = 1$.
(c) Find the speed at $t = 1$.

5 If the curve C is given by $x = 3t^2 + 1, y = 4t, z = e^{t-1}$, find an equation of the tangent line at the point $P(4, 4, 1)$.

6 If $\mathbf{u}(t) = t^2\mathbf{i} + 6t\mathbf{j} + t\mathbf{k}$ and $\mathbf{v}(t) = t\mathbf{i} - 5t\mathbf{j} + 4t^2\mathbf{k}$ find
(a) the values of t for which $\mathbf{u}(t)$ and $\mathbf{v}'(t)$ are orthogonal.
(b) $D_t[\mathbf{u}(t) \times \mathbf{v}(t)]$.
(c) $D_t[\mathbf{u}(t) \cdot \mathbf{v}(t)]$.
(d) $\int \mathbf{u}(t)\, dt$.
(e) $\int_0^1 \mathbf{v}(t)\, dt$.

7 Find $\mathbf{u}(t)$ if $\mathbf{u}'(t) = e^{-t}\mathbf{i} - 4\sin 2t\mathbf{j} + 3\sqrt{t}\mathbf{k}$ and $\mathbf{u}(0) = -\mathbf{i} + 2\mathbf{j}$.

Verify the identities in Exercises 8 and 9 without using components.

8 $D_t|\mathbf{u}(t)|^2 = 2\mathbf{u}(t) \cdot \mathbf{u}'(t)$

9 $D_t(\mathbf{u}(t) \cdot \mathbf{u}'(t) \times \mathbf{u}''(t)) = \mathbf{u}(t) \cdot \mathbf{u}'(t) \times \mathbf{u}'''(t)$

In Exercises 10–12, find the curvature of the given curve at the point P.

10 $y = xe^x$; $P(0, 0)$

11 $x = 1/(1 + t), y = 1/(1 - t)$; $P(2/3, 2)$

12 $x = 2t^2, y = t^4, z = 4t$; $P(x, y, z)$

In Exercises 13 and 14, find the tangential and normal components of acceleration at time t if the position vector of a particle is as indicated.

13 $\mathbf{r}(t) = \sin 2t\mathbf{i} + \cos t\mathbf{j}$

14 $\mathbf{r}(t) = 3t\mathbf{i} + t^3\mathbf{j} + t\mathbf{k}$

15 Extrema of functions of two variables

16 Lagrange multipliers

Exercises

In Exercises 1–4 determine the domain of the function f.

1 $f(x, y) = \sqrt{36 - 4x^2 + 9y^2}$

2 $f(x, y) = \ln xy$

3 $f(x, y, z) = (z^2 - x^2 - y^2)^{-3/2}$

4 $f(x, y, z) = \sec z/(x - y)$

In Exercises 5–10 find the first partial derivatives of f.

5 $f(x, y) = x^3 \cos y - y^2 + 4x$

6 $f(r, s) = r^2 e^{rs}$

7 $f(x, y, z) = (x^2 + y^2)/(y^2 + z^2)$

8 $f(u, v, t) = u \ln (v/t)$

9 $f(x, y, z, t) = x^2 z\sqrt{2y + t}$

10 $f(v, w) = v^2 \cos w + w^2 \cos v$

In Exercises 11 and 12 find the second partial derivatives of f.

11 $f(x, y) = x^3y^2 - 3xy^3 + x^4 - 3y + 2$

12 $f(x, y, z) = x^2 e^{y^2 - z^2}$

13 If $u = (x^2 + y^2 + z^2)^{-1/2}$ prove that $\partial^2 u/\partial x^2 + \partial^2 u/\partial y^2 + \partial^2 u/\partial z^2 = 0$.

14 (a) Find dw if $w = y^3 \tan^{-1} x^2 + 2x - y$. (*b*) Find dw if $w = x^2 \sin yz$.

15 Find Δw and dw if $w = x^2 + 3xy - y^2$. Use Δw to find the exact change and dw to find the approximate change in w if (x, y) changes from $(-1, 2)$ to $(-1.1, 2.1)$.

16 Ohm's Law states that $R = E/I$. If measurements are $E = 108$ volts and $I = 2$ amperes with a possible error of 0.2 volts and 0.01 amperes, respectively, use differentials to approximate the maximum error in the calculated value of R (in ohms).

Use the Chain Rule to find the solutions in Exercises 17–19.

17 If $s = u^2v + v^2w - w^3u$, $u = y\cos x$, $v = xe^{-y}$, and $w = y \ln x$, find $\partial s/\partial x$ and $\partial s/\partial y$.

18 If $z = \sqrt{x^2 + y^2}$, $x = re^{2st}$, and $y = tr^2e^{-s}$, find $\partial z/\partial r$, $\partial z/\partial s$, and $\partial z/\partial t$.

19 If $w = x \tan y + y \tan z$, $x = t^3$, $y = e^{-2t}$, and $z = 1/t^2$, find dw/dt.

20 Find the directional derivative of $f(x, y) = 3x^2 - y^2 + 5xy$ at the point $P(2, -1)$ in the direction of the vector $\mathbf{a} = -3\mathbf{i} - 4\mathbf{j}$. What is the maximum rate of increase of $f(x, y)$ at P?

21 The temperature at the point (x, y, z) is given by $T(x, y, z) = 3x^2 + 2y^2 - 4z$. Find the rate of change of T at the point $P(-1, -3, 2)$ in the direction from P to the point $Q(-4, 1, -2)$.

22 A curve C is given parametrically by $x = t$, $y = t^2$, $z = t^3$. If $f(x, y, z) = y^2 + xz$, find $D_{\mathbf{u}} f(2, 4, 8)$, where $\mathbf{u}$ is a unit tangent vector to C at $P(2, 4, 8)$.

23 Find equations of the tangent plane and normal line to the graph of $7z = 4x^2 - 2y^2$ at the point $P(-2, -1, 2)$.

24 Show that every plane tangent to the cone $x^2/a^2 - y^2/b^2 + z^2/c^2 = 0$ passes through the origin.

25 If $y = f(x)$ satisfies the equation $x^3 - 4xy^3 - 3y + x - 2 = 0$, use partial derivatives to find $f'(x)$.

26 If $z = f(x, y)$ satisfies the equation $x^2y + z\cos y - xz^3 = 0$, find $\partial z/\partial x$ and $\partial z/\partial y$.

27 Find the extrema of f if $f(x, y) = x^2 + 3y - y^3$.

28 The material for the bottom of a rectangular box costs twice as much per square inch as that for the sides and top. What relative dimensions will minimize the cost if the volume is fixed?

29 Given $f(x, y) = x^2/4 + y^2/25$, sketch several level curves associated with f and represent $\nabla f]_P$ by a vector for a point P on each curve.

30 Given $F(x, y, z) = z + 4x^2 + 9y^2$, sketch several level surfaces associated with F and represent $\nabla F]_P$ by a vector for a point P on each surface.

31 Find the local extrema of $f(x, y, z) = xyz$ subject to the constraint $x^2 + 4y^2 + 2z^2 = 8$.

32 Use Lagrange multipliers to find the local extrema of $f(x, y, z) = 4x^2 + y^2 + z^2$ subject to the constraints $2x - y + z = 4$ and $x + 2y - z = 1$. Check your answer using single variable methods.

33 Find the points on the graph of $1/x + 2/y + 3/z = 1$ which are closest to the origin.

34 A hopper in a grain elevator has the shape of a right circular cone of radius 2 ft, surmounted by a right circular cylinder. If the volume is $100\,\text{ft}^3$, find the altitudes h and k of the cylinder and cone, respectively, that will minimize the curved surface area.

Exercises

Evaluate the integrals in Exercises 1–6.

1 $\int_{-1}^{0} \int_{x+1}^{x^3} (x^2 - 2y)\, dy\, dx$

2 $\int_{1}^{2} \int_{1}^{\ln y} \frac{1}{y}\, dx\, dy$

3 $\int_{0}^{3} \int_{r}^{r^2+1} r\, d\theta\, dr$

4 $\int_{2}^{0} \int_{0}^{z^2} \int_{x}^{z} (x + z)\, dy\, dx\, dz$

5 $\int_{0}^{2} \int_{\sqrt{y}}^{1} \int_{z^2}^{y} xy^2 z^3\, dx\, dz\, dy$

6 $\int_{0}^{\pi/2} \int_{0}^{\pi/4} \int_{0}^{a\cos\phi} \rho^2 \sin\phi\, d\rho\, d\phi\, d\theta$

In each of Exercises 7–10 express $\iint_R f(x, y)\, dA$ as an iterated integral if R is the region bounded by the graphs of the given equations.

7 $x^2 - y^2 = 4, x = 4$

8 $x^2 - y^2 = 4, y = 4, y = 0$

9 $y^2 = 4 + x, y^2 = 4 - x$

10 $y = -x^2 + 4, y = 3x^2$

Each of the integrals in Exercises 11 and 12 represents the area of a region R in the xy-plane. Describe R.

11 $\int_{-1}^{1} \int_{e^y}^{y^3} dx\, dy$

12 $\int_{-1}^{0} \int_{x}^{-x^2} dy\, dx$

In Exercises 13 and 14 reverse the order of integration and evaluate the resulting integral.

13 $\int_{0}^{3} \int_{y^2}^{9} ye^{-x^2}\, dx\, dy$

14 $\int_{0}^{1} \int_{x}^{\sqrt{x}} e^{x/y}\, dy\, dx$

In Exercises 15 and 16 find the mass and the center of mass of the lamina which has the shape of the region bounded by the graphs of the given equations and having the indicated density.

15 $y = x, y = 2x, x = 3$; density at the point $P(x, y)$ is directly proportional to the distance from the y-axis to P.

16 $y^2 = x, x = 4$; density at the point $P(x, y)$ is directly proportional to the distance from the line with equation $x = -1$ to P.

17 A lamina has the shape of the region which lies inside the graph of $r = 2 + \sin\theta$ and outside the graph of $r = 1$. Find the mass if the density at the point $P(r, \theta)$ is inversely proportional to the distance from the pole to P.

18 Find the area of the region bounded by the polar axis and the graphs of $r = e^\theta$ and $r = 2$ from $\theta = 0$ to $\theta = \ln 2$.

19 Use polar coordinates to evaluate

$$\int_{-a}^{0} \int_{-\sqrt{a^2 - x^2}}^{0} \sqrt{x^2 + y^2}\, dy\, dx.$$

20 Find I_x, I_y, and I_O for the lamina bounded by the graphs of $y = x^2$ and $y = x^3$ if the density at the point $P(x, y)$ is directly proportional to the distance from the y-axis to P.

21 A homogeneous lamina has the shape of a right triangle with sides of lengths a, b, and $\sqrt{a^2 + b^2}$. Find the moment of inertia and the radius of gyration with respect to a line along the side of length a.

22 A lamina has the shape of the region between concentric circles of radii a and b, where $a < b$. If the density at a point P is directly proportional to the distance from the center to P, use polar coordinates to find the moment of inertia with respect to a line through the center.

23 Find the volume of the solid which lies under the graph of $z = xy^2$ and over the rectangle in the xy-plane with vertices $(1, 1, 0)$, $(2, 1, 0)$, $(1, 3, 0)$, and $(2, 3, 0)$.

24 Express $\iiint_Q f(x, y, z)\,dV$ as an iterated integral in six different ways, where Q is bounded by the graphs of $y = x^2 + 4z^2$ and $y = 4$.

25 Use triple integrals to find the volume and centroid of the solid bounded by the graphs of $z = x^2$, $z = 4$, $y = 0$, and $y + z = 4$.

26 Set up a triple integral for the moment of inertia with respect to the z-axis of the solid bounded by the graphs of $z = 9x^2 + y^2$ and $z = 9$ if the density at the point $P(x, y, z)$ is inversely proportional to the square of the distance from the point $(0, 0, -1)$ to P.

27 Set up a triple integral for the moment of inertia with respect to the y-axis of the solid bounded by the graphs of $x^2 - y^2 + z^2 = 1$, $y = 0$, and $y = 4$ if the density at the point $P(x, y, z)$ is directly proportional to the distance from the y-axis to P.

28 A homogeneous solid is bounded by the graphs of $z = 9 - x^2 - y^2$, $x^2 + y^2 = 4$, and $z = 0$. Use cylindrical coordinates to find the following.
(a) The mass
(b) The center of mass
(c) The moment of inertia with respect to the z-axis

29 A solid has the shape of a sphere of radius a. Use spherical coordinates to find the mass if the density at a point P is directly proportional to the distance from the center to P.

30 Find the surface area of that part of the cone $z = (x^2 + y^2)^{1/2}$ which is inside the cylinder $x^2 + y^2 = 4x$.