2.1 Quadratic Equations

Quadratic Eqn: \( ax^2 + bx + c = 0 \) \( a \neq 0 \)

Zero Product Property

For \( a, b \in \mathbb{R}, ab = 0 \Rightarrow a = 0 \) or \( b = 0 \),
or \( a = b = 0 \).

4 ways to solve quadratic eqns

1. Square root technique (only works if \( b = 0 \))
2. Factoring
3. Completing the square
4. Quadratic formula: \( x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \)

Derive quadratic formula:)

\( ax^2 + bx + c = 0 \)
Ex 1  Solve

(a) $25x^2 - 49 = 0$

(b) $x^2 - 4x = 3x^2$
Ex. 2

(a) \( x^2 + 17x = 8x - 14 \)

(b) \( x^2 + 2x + 4 = 0 \)

(c) \( (x+1)^2 = 2 \)
Ex 3  Solve.

(a) \((y - 2)^2 - 5(y - 2) - 24 = 0\)

(b) \[\frac{5}{t + 4} - \frac{3}{t - 2} = 4\]
2.2 Quadratic Functions: Parabolas

Quadratic Fn => \( y = f(x) = ax^2 + bx + c \)
(a quadratic eqn in two variables)
when we graph all the solutions to this, the points form a parabola.

For \( y = ax^2 + bx + c \),
if \( a > 0 \), \( \cup \) concave up
if \( a < 0 \), \( \cap \) concave down

axis of symmetry

Let's figure out where the vertex is.
(algebraically) so we can always find it.

if we plug in \( x = 0 \) we get pt on y-axis =
\( x = 0 \Rightarrow y = a(0^2) + b(0) + c \) (= \( y = c \)), i.e. parabola
goes thru \((0, c)\)

We can see, by symmetry of parabola, that there is another pt whose y-value is c.
2.2 (cont)

\[ C = ax^2 + bx + c \quad (\Rightarrow) \quad 0 = ax^2 + bx \]

\[ 0 = x(ax + b) \]

\[ x = 0, \quad \text{or} \quad ax + b = 0 \]

\[ ax = -b \]

\[ x = -\frac{b}{a} \]

You can see (from symmetry) that the x-value of the vertex is halfway between 0 and \(-\frac{b}{a}\), i.e. \[ x = -\frac{b}{2a} \].

\[ \text{vertex at } \left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right) \]

\[ \text{Axis of Symmetry} \]

\[ x = -\frac{b}{2a} \]

Ex 1 For \( y = -2x^2 - 4x + 6 \)

(a) Find vertex

(b) Is the vertex a min or max pt?
2.2 (Cont.)

Ex 2 For \( y = x^2 - 6x + 9 \),

(a) find vertex.
(b) Is it a min or max pt?
(c) find zeroes of graph.
(d) Sketch the graph.
Ex 3. Describe shifting for $y = (x-10)^2 + 1$.

Ex 4. Find the average rate of change of $y = \frac{1}{2}x^2 + 3x + 8$ between $x=2$ and $x=4$. 
Ex 5 If 100 ft of fencing is used to enclose a rectangular yard, then the resulting area is given by \( A = x(50-x) \). Graph this equation and give the length and width that maximize area.
Supply Demand + Market Equilibrium

Ex 1. If the supply function for a commodity is \( p = q^2 + 8q + 20 \) and the demand function is \( p = 100 - 4q - q^2 \), find the equilibrium quantity and equilibrium price. (Sketch both curves.)
2.3 (cont)

Ex 2 For the last example, if an $8 \text{ tax}$ is placed on production & passed through the suppliers, find the new equilibrium pt.
Break-Even Points and Maximization

Ex 3 If a company has total costs $C(x) = 1600 + 1500x$ and total revenue is $R(x) = (1600 - x)x$, find the break even pts.

Break Even pts occur when $R(x) = C(x)$

$\Rightarrow P(x) = 0$
2.3 (cont.)

Ex 4 Find maximum revenue given
\[ R(x) = 1600x - x^2. \]

Ex 5 Suppose a company has fixed costs of $300
and variable costs of \( \frac{3}{4}x + 1440 \) dollars per unit,
where \( x \) = total # units produced. Suppose further
that its selling price is \( 1500 - \frac{1}{4}x \) dollars per unit.
(a) Find break even pts.
(b) Find max revenue

(c) Find max profit, and price that yields it.
2.4 Special Functions and Their Graphs

**Polynomials**

\[ y = mx + b \]

\[ y = ax^2 + bx + c \]

\[ y = ax^3 + bx^2 + cx + d \]

**Radical**

\[ y = \sqrt{x} \]

\[ y = \sqrt[3]{x} \]

**Powers**

\[ y = x^b \]

\[ y = x^1 \]

\[ y = x^b \]

\(0 < b < 1\)
2.4 (cont)

**Shifting Graphs**

\[ y = f(x+h) + k \]  
(assume \( h, k > 0 \))  
shifts up by \( k \) units  
and left by \( h \) units

Ex 1  Describe shifting of \( f(x) = (x-2)^2 + 3 \)  
compared to base graph \( y = x^2 \).

**Piecewise Functions**

\[ f(x) = |x| = \begin{cases} 
  x & \text{if } x \geq 0 \\
  -x & \text{if } x < 0 
\end{cases} \]

Ex 2  Graph  
\[ f(x) = \begin{cases} 
  x^2 + 5 & \text{if } x \geq 2 \\
  x + 3 & \text{if } 0 \leq x < 2 \\
  -x - 1 & \text{if } x < 0 
\end{cases} \]
2.4 (cont)

Rational Functions = ) \[ \frac{f(x)}{g(x)} \]
where \( f(x) \) and \( g(x) \) are both polynomials.

Asymptotes

\( 1 \) vertical \( \Rightarrow \) "restriction" of what \( x \)-values graph cannot have (comes from domain)

\( 2 \) horizontal \( \Rightarrow \) "description" of what happens to graph as \( x \to \pm \infty \)

Ex 3 Graph \( f(x) = \frac{x-3}{x+2} \)
Ex 4 Given function \[ y = \begin{cases} \frac{1}{2}x + 4 & x < 0 \\ 4 - x & 0 \leq x < 4 \\ 0 & x \geq 4 \end{cases} \]

(a) find \( y (1) \)

(b) find \( y (3.9) \)

(c) find \( y (-4) \)

Ex 5  Graph \[ y = (x+2)^3 - 3 \]
2.6 Composite and Inverse Functions

**Composite Functions**

Given $f(x)$ and $g(x)$, 
$$ (f \circ g)(x) = f(g(x)) $$

**Example 1**

Given $f(x) = 2x + 8$, 
$$ g(x) = \frac{1}{x^3} $$

(a) find $(f \circ g)(x)$.

(b) find $(g \circ f)(x)$.

**Example 2**

Find for $f(x)$ and $g(x)$, 
$$ (f \circ g)(x) = \frac{1}{5x^3 + 4} $$
Inverse Functions

An inverse function basically "undoes" what original function did to input notation: \( f^{-1}(x) \) (read "f inverse of x")

\[
= f(f^{-1}(x)) = x = f^{-1}(f(x))
\]

Ex 3 Are \( f(x) = 5x - 1 \) and \( g(x) = \frac{x + 1}{5} \) inverse functions?

Finding an inverse function \( f^{-1} \) the operations basically you need to undo the operations in the opposite order to how they were done.
2.6 (cont)

Ex 4: Find the inverse function for

\[ y = \frac{(x-2)^3}{4} + 5 \]

Does every function have an inverse?

A function has an inverse if it passes the horizontal line test! (i.e., if it is one-to-one → every input has exactly one output & every output has also only one input.)
2.6 (cont)

inverse pts are mirror images across line \( y = x \).

**Ex 5** Does \( y = x^2 \) have inverse function?

**Ex 6** Is function defined by \( \{(1,3), (6,2), (4,3)\} \) one-to-one?

**Ex 7** To convert from Celsius to Fahrenheit, you can use \( F = \frac{9}{5}C + 32 \). Change this formula to allow you to convert from °F to °C.