Problem 1 (Fractions). Simplify:

$$\text{LCM} = 15 \quad \left( \frac{\frac{3}{5} - \frac{2}{3}}{\frac{1}{3} + \frac{3}{5}} \right) \frac{15}{15}$$

$$= \frac{\frac{3}{5}(15) - \frac{2}{3}(15)}{\frac{1}{3}(15) + \frac{3}{5}(15)} = \frac{9 - 10}{5 + 9} = \frac{-1}{14}$$

Problem 2 (A Linear Equation). Solve the equation

$$4x - 3 = 10 - 2(x - 1)$$

$$4x - 3 = 10 - 2x + 2$$

$$4x - 3 = 12 - 2x$$
$$+2x \quad +3 \quad +3 \quad +2x$$

$$6x = 15$$

$$x = \frac{15}{6} = \frac{5}{2}$$
Problem 3 (A Quadratic Equation). Find all solutions of the equation

\[ x^2 - x - 20 = 0 \]

\[ (x - 5)(x + 4) = 0 \]

\[ x - 5 = 0 \quad \text{or} \quad x + 4 = 0 \]

\[ x = 5 \quad \text{or} \quad x = -4 \]

\[ x = 5, -4 \]

Problem 4 (Another Quadratic Equation). Find all solutions of the equation

\[ x^2 - 2x - 5 = 0 \]

\[ a = 1 \quad b = -2 \quad c = -5 \]

\[ x = \frac{2 \pm \sqrt{4 - 4(1)(-5)}}{2(1)} \]

\[ x = \frac{2 \pm \sqrt{4 + 20}}{2} = \frac{2 \pm 2\sqrt{6}}{2} \]

\[ x = \frac{2 \pm 2\sqrt{6}}{2} = \frac{2(1 \pm \sqrt{6})}{2} \]

\[ x = 1 \pm \sqrt{6} \]
Problem 5 (A rational equation). Find all solutions of

\[ L(x) = \frac{(x-2)(x-3)}{x-2} \quad \frac{8}{x-2} - \frac{5}{x-3} + 1 = 0 \quad x\neq 2,3 \]

\[
\frac{(x-2)(x-3)8}{(x-2)} - \frac{5(x-2)(x-3)}{(x-3)} + 1 (x-2)(x-3)= 0(x-2)(x-3)
\]

\[ 8(x-3) - 5(x-2) + (x-1)(x-3) = 0 \]

\[ 8x - 24 - 5x + 10 + x^2 - 2x - 3x + 6 = 0 \]

\[ x^2 - 2x - 8 = 0 \]

\[ (x-4)(x+2) = 0 \]

\[ x = 4, -2 \]

Problem 6 (Polynomials). Write the following polynomial expression in standard form. What is its degree and its leading coefficient?

\[ (x^2 - 1)(x + 3) + 2x + 4 \]

\[ = x^3 + 3x^2 - x - 3 + 2x + 4 \]

\[ = x^3 + 3x^2 + x + 1 \]

Degree = 3
Leading coefficient = 1
For the next three questions let

\[
f(x) = \frac{x - 1}{x^2 - 2x} = \frac{x - 1}{x(x - 2)}
\]

**Problem 7 (Domain).** What is the natural domain of \(f\)?

\[X \in \mathbb{R}, \quad X \neq 0, 2\]

**Problem 8 (Evaluate at a point).** Find \(f(5)\).

\[
f(5) = \frac{5 - 1}{5^2 - 2(5)} = \frac{4}{25 - 10} = \frac{4}{15}
\]

**Problem 9 (Evaluate at an expression).** Find \(f(2x + 1)\) and express it in the standard form of a rational expression.

\[
f(2x+1) = \frac{2x+1-x}{(2x+1)^2-2(2x+1)}
\]

\[
= \frac{x+1}{4x^2 + 4x + 1 - 4x - 2}
\]

\[
= \frac{x+1}{4x^2 - 1}
\]


Problem 10 (Radical Equations). Solve the equation
\[
\sqrt{x+4} + \sqrt{x+11} = 7.
\]
\[
\left(\sqrt{x+11}\right)^2 = \left(7-\sqrt{x+4}\right)^2
\]
\[
x+11 = 49-14\sqrt{x+4} + x+4
\]
\[
x=53-14\sqrt{x+4}
\]
\[
\frac{-42}{-14} = \frac{-14\sqrt{x+4}}{-14}
\]
\[
3 = \sqrt{x+4} \quad \Rightarrow \quad q = x+4
\]
\[
5 = x
\]

Problem 11 (Linear System). Solve the system
\[
4x - y = 1
\]
\[
2x + y = 0
\]
Show all your work. Don’t just give the answer.

\[
\begin{align*}
4x - y &= 1 \\
+ 2x + y &= 0
\end{align*}
\]
\[
6x = 1
\]
\[
x = \frac{1}{6}
\]
\[
4\left(\frac{1}{6}\right) - y = 1 \quad \Rightarrow \quad \frac{2}{3} - y = 1
\]
\[
-\frac{2}{3} - y = \frac{2}{3}
\]
\[
y = \frac{1}{3}
\]
\[
5 \quad \Rightarrow \quad y = -\frac{1}{3}
\]
Problem 12 (Another Linear System). Solve the linear system

\[
\begin{align*}
\begin{cases}
\ell_1 : & x + y + z = 5 \\
\ell_2 : & x + 2y - z = 3 \\
\ell_3 : & 2x + 3y - z = 5
\end{cases}
\end{align*}
\]

Again, show all your work, don’t just give the answer.

\[
\begin{align*}
(1) \quad & x + y + z = 5 \\
(2) \quad & y - 2z = -2 \\
(3) \quad & y - 3z = -5
\end{align*}
\]

\[
\begin{align*}
x + y + z &= 5 \\
y - 2z &= -2 \\
-3 &= -3
\end{align*}
\]

\[
\begin{align*}
z &= 3 \\
y - 2(3) &= -2 \\
y - 6 &= -2 \\
y &= 4
\end{align*}
\]

\[
\begin{align*}
x + y + z &= 5 \\
x + 4 + 3 &= 5 \\
x + 7 &= 5 \\
x &= -2
\end{align*}
\]

\((-2, 4, 3)\)
Problem 13 (Straight Lines). Find an equation of the line that passes through (2,1) and has slope -1/2. Draw its graph.

\[ m = -\frac{1}{2} \quad (2, 1) \]

\[ y - 1 = -\frac{1}{2} (x - 2) \]

\[ y - 1 = -\frac{1}{2} x + 1 \]

\[ y = -\frac{1}{2} x + 2 \]

Problem 14 (Distance). Find the distance between the points (-1,3) and (2,4).

\[ d = \sqrt{(3-4)^2 + (-(-2))^2} \]

\[ = \sqrt{(-1)^2 + (-3)^2} \]

\[ = \sqrt{1 + 9} \]

\[ = \sqrt{10} \]
Problem 15 (Powers). Simplify (i.e., write with only positive exponents, such that $x$ and $y$ occur only once) the expression

$$\left(\frac{x^2y^{-3}}{(x^{-1}y)^{-3}}\right)^2 = \frac{x^4}{x^3y^{-3}} = \frac{y^3}{x^3}$$

Problem 16 (Radical Expressions). Simplify the expression

$$\left(\frac{x^{3/2}}{\sqrt[1/6]{x}}\right)^{2/5}$$

and write it as a power with a single exponent.

$$\left(\frac{\frac{3}{2} - \frac{1}{6}}{6}\right)^{15}$$

$$\left(\frac{\frac{8}{3}}{6}\right)^{15}$$

$$= \frac{8}{15}$$
Problem 17 (Rational Expressions). Simplify the following expression

\[
L \cdot C = \frac{(x+1)(x-2)(x-3)}{x+1} + \frac{1}{x-2} - \frac{2}{x-3}
\]

\[
= \frac{2 \cdot \frac{(x-2)(x-3)}{x+1}}{x+1} + \frac{1}{x-2} \cdot \frac{(x+1)(x-3)}{x+1} - \frac{2}{x-3} \cdot \frac{(x+1)(x-3)}{x+1}
\]

\[
= \frac{2(x^2-5x+6) + x^2-2x-3 - 2(x^2-x-3)}{(x+1)(x-2)(x-3)}
\]

\[
= \frac{2x^2-10x+12 + x^2-2x-3 - 2x^2+2x+6}{(x+1)(x-2)(x-3)}
\]

Problem 18 (Inequality). Solve the following inequality

\[
|2x - 1| + 2 \leq 8
\]

\[
|2x - 1| \leq 6
\]

\[
-6 \leq 2x - 1 \leq 6
\]

\[
-5 \leq 2x \leq 7
\]

\[
-\frac{5}{2} \leq x \leq \frac{7}{2}
\]
Problem 19 (Word Problem). You are at a store buying some fencing for your new garden. You would like it to have an area of 36 square meters, and to be in a shape of a rectangle. The store clerk cut 26 meters of chicken wire. How long will the sides of your garden be?

\begin{align*}
\rho &= 2x + 2y = 26 \\
A &= xy = 36 \text{ m}^2 \\
(1) \quad 2x + 2y &= 26 \
\Rightarrow \quad x + y &= 13 \
\Rightarrow \quad y &= 13 - x \\
(2) \quad xy &= 36 \
\Rightarrow \quad x(13 - x) &= 36 \\
13x - x^2 &= 36 \\
0 &= x^2 - 13x + 36 \\
\text{Answer:} \quad 9 \text{ m by } 4 \text{ m}
\end{align*}

Problem 20 (Another Word Problem). You obtain 5 gallons of 14% vinegar solution by mixing suitable amounts of 5% vinegar solution and of 20% vinegar solution. How much of each type of vinegar solution do you use?

\begin{align*}
\text{A} & \quad 5 \text{ g} \times 0.05x \\
\text{B} & \quad 20 \text{ g} \times 0.2(5-x) \\
\text{mix} & \quad 14 \text{ g} \times 0.14(5) \\
0.05x + 0.2(5-x) &= 0.14(5) \\
100(0.05x + 1 - 0.2x) &= (0.7)100 \\
5x + 100 - 20x &= 70 \\
-15x &= -30 \\
x &= 2
\end{align*}

Answer: 2 gallons of 5% solution, 3 gallons of 20% solution.