Exercise 4: A flower bed will be in the shape of a sector of a circle (a pre-shaped region) of radius \( r \) and vertex angle \( \theta \). Find \( r \) and \( \theta \) if its area is a constant \( A \) and the perimeter is a minimum.}

\[ \begin{align*}
A &= \frac{\theta}{2\pi} (\pi r^2) \\
A &= \frac{\theta r^2}{2}
\end{align*} \]

\( A \) is a constant, so, solve for \( \theta \):

\[ \theta = \frac{2A}{r^2} \]

Test:

\[ r = \frac{\sqrt{A}}{2}, \quad 2 - \frac{2A}{4} = 2 - 0 = 2 > 0 \]

\[ r = 2\sqrt{A}, \quad 2 - \frac{2A}{4\sqrt{A}} > 0 \]

\( r = \sqrt{A} \) and

\[ \theta = \frac{2A}{(\sqrt{A})^2} = \frac{2A}{A} = 2 \]
Ex 6 A farmer has 80 ft of fence. He needs to enclose three identical pens along one side of his barn (the side along the barn needs no fence). What dimensions for the total enclosure make the area of the pens as large as possible?

\[ \text{Perimeter of fence} \quad P = 4x + y = 80 \text{ ft} \quad \text{(given)} \]

\[ y = 80 - 4x \]

\[ A = xy = x(80 - 4x) \]

\[ A = 80x - 4x^2 \]
\[ A' = -8x + 80 = 0 \]
\[ 8x = 80 \]
\[ x = 10 \text{ ft} \]

\[ \Rightarrow y = 80 - 4(10) = 40 \]

10 ft by 40 ft