Math5900 Project #2
Algebraic Thinking

Name: _____

Due date: _____

Show all your work for these problems and communicate your answers thoroughly. Please be neat so my grading time is somewhat pleasurable. :)

Part 1 (Thinking)

Read the two articles provided in this packet, and answer these questions. This part must be typed!! Please give thoughtful answers that have some depth.

1. What would you define as algebraic thinking?

2. When do you think students begin to think algebraically? Explain.

3. How has this changed or expanded your view of algebra?

Part 2 (Teaching)

Choose two lessons, from two different chapters, in your Navigating Through Algebra book (the one that's appropriate for your grade) to teach to your class.   After teaching each lesson, type a report answering the following questions. (The report for each lesson should be around two pages. Please don't give me a dissertation. :))

4. What insights did the students have regarding the mathematics?

5. What further questions did the lesson bring up in class?

6. How did it further the children's algebraic thinking?

7. What activity did you follow it up with to deepen their mathematical understanding?

Part 3 (Doing)

For this section, do the following problems. For each problem, (i) write up your solution, showing all your work, and (ii) write about how algebra was used in your solution process.

8. **The Thousand Locker Problem**: Imagine you are at a school that has student lockers. There are 1000 lockers, all shut and unlocked, and 1000 students.
Here's the problem:

1. Suppose the first student goes along the row and *opens every locker.*

2. The second student then goes along and *shuts every other locker* beginning with **number 2**.

3. The third student *changes the state* of every **third** locker beginning with **number 3**. (If the locker is open the student shuts it, and if the locker is closed the student opens it.)

4. The fourth student *changes the state* of every **fourth** locker beginning with **number 4**.

Imagine that this continues until the thousand students have followed the pattern with the thousand lockers. At the end, which lockers will be open and which will be closed? **Why?**

9. **Fun with Calendars**: Take *any* calendar. Tell your friend to choose 4 days that form a square like the four below. Your friend should tell you only the sum of the four days, and you can tell her what the four days are.

How does the puzzle work?   (You know how people always want to see a use for algebra? Well this puzzle uses algebra. :))

7	8
14	15

10. **Clock Challenge**: A grandfather clock at my in-laws is known to lose 7 minutes every hour. On a Sunday, about a month ago I noticed that it was EXACTLY three minutes slow- to the second. Later that month, and much to my surprise, the clock was EXACTLY at the right time. My father-in-law remarked that he had done nothing to adjust the clock, and I knew he hadn't. Figure out what day of the week was my second visit.

11. **Calendar Problem**:   Using the numbers 1 9 9 2 in a "locked" position, can you develop a 31 day calendar for the month of October? You can use addition (+), subtraction (-), multiplication (*), division (/) exponents (^) factorial (!) square root ($\sqrt{}$) and, naturally, parentheses ( ).

Example: The 13th of the month could be: $(1+\sqrt{9})!-9-2$.
          (Notice that the numbers appear in the "locked" sequence)

Day	Formula
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	
16	
17	
18	
19	
20	
21	
22	
23	
24	
25	
26	
27	

Day	Formula
28	
29	
30	
31	

12. **Traffic Jam**
Here's the problem:

There are seven stepping stones and six people. On the three lefthand stones, facing the center, stand three of the people. The other three people stand on the three righthand stones, also facing the center. The center stone is not occupied.

The challenge: exchanging places

Everyone must move so that the people originally standing on the righthand stepping stones are on the lefthand stones, and those originally standing on the lefthand stepping stones are on the righthand stones, with the center stone again unoccupied.

The rules:

1. After each move, each person must be standing on a stepping stone.
2. If you start on the left, you may only move to the right. If you start on the right, you may only move to the left.
3. You may "jump" another person if there is an empty stone on the other side. You may not "jump" more than one person.
4. Only one person can move at a time.

13. During the next few weeks, be aware of times you need algebraic thinking in your daily life to work out a solution to a problem. Document two such problems here, along with your solution.

(a)

(b)

14. Multiply out the following expressions (the first couple are started for you). Do you recognize a pattern with the coefficients?

$$(x+y)^1 = x+y$$
$$(x+y)^2 = x^2 + 2xy + y^2$$
$$(x+y)^3 = x^3 + 3x^2 y + 3xy^2 + y^3$$
$$(x+y)^4 =$$
$$(x+y)^5 =$$

Use your pattern to determine the expanded (multiplied out) version of

(a) $(x-y)^{10}$

(b) $(2a-3)^5$