

Math5900 Final Review

1. Find the formula to predict any term in this sequence and then find the 425th number in this sequence

-3, 3, 9, 15, ...

Answer: either $a_n = 6n - 3$ if you start counting at $n=0$ OR

$a_n = 6n - 9$ if you start counting at $n=1$

The 425th number is $6(425) - 9$ [or $6(424) - 3$] which is 2541

2. Write a mathematically convincing argument to explain the formula

$$1+2+3+\dots+n = \frac{n(n+1)}{2}$$

(We've done this a zillion times...I'm sure you have it in your notes.)

3. Convert these numbers to a different base as indicated.

(a) 2304_5 to base 10 Answer: $2(125) + 3(25) + 0(5) + 4(1) = 329$

(b) 762_8 to base 10 Answer: $7(64) + 6(8) + 2(1) = 498$

(c) 143 to base 3 Answer: $143 = 1(81) + 2(27) + 0(9) + 2(3) + 2(1) = 12021_3$

(d) 111100001_2 to base 10 Answer: $111100001_2 = 1(256) + 1(128) + 1(64) + 1(32) + 0(16) + 0(8) + 0(4) + 0(2) + 1(1) = 481$

(e) 419 to base 6 Answer: $419 = 1(216) + 5(36) + 3(6) + 5(1) = 1535_6$

5. Draw a Venn Diagram for all the number systems (including fractions) and place these numbers in the proper place in the Venn Diagram.

(a) 21% Answer: **F**

(b) $\frac{4}{4}$ Answer: **N**

(c) -20 Answer: **Z**

(d) $\frac{1}{3}$ Answer: **F**

(e) 0 Answer: **W**

(f) -3.16 Answer: **Q**

(g) 6.090090009... Answer: **I** (Irrational number)

(h) $\frac{-5}{9}$ Answer: **Q**

(i) 8 Answer: **N**

(j) 3π Answer: **I**

(k) $5.\bar{9}$ Answer: **N**

(l) $\frac{-14}{7}$ Answer: **Z**

- (m) $\sqrt{169}$ Answer: **N**
 (n) $\sqrt{7}$ Answer: **I**
 (o) 0.76 Answer: **F**
 (p) $2.11\overline{34}$ Answer: **F**

6. Order these numbers and then place them on a number line.

- (a) $\frac{3}{40}=0.075$
 (b) $69\% = 0.69$
 (c) $\frac{8}{5}=1.6$
 (d) 0.5
 (e) 0.2
 (f) $\frac{1}{4}=0.25$
 (g) $1.\overline{9}=2$
 (h) 0
 (i) $\frac{2}{3}=0.\overline{6}$
 (j) 1.6

Answer (in order): h, a, e, f, d, i, b, c&j, g

6. Convert these numbers into a fraction (or mixed number).

- (a) $24.1\overline{35}=24\frac{134}{990}=24\frac{67}{495}$
 (b) $46\% = \frac{46}{100}=\frac{23}{50}$
 (c) $5.113\overline{4}=5\frac{1021}{9000}$

7. Simplify these expressions.

- (a) $2\frac{3}{5}\div\frac{2}{15}=\frac{13}{5}\cdot\frac{15}{2}=\frac{13\cdot3}{2}=\frac{39}{2}$
 (b) $5^45^{-2}5^0(5^2)^3=5^45^{-2}5^6=5^8$
 (c) $\frac{1}{7}\div\frac{6}{14}\cdot3\frac{3}{4}=\frac{1}{7}\cdot\frac{14}{6}\cdot\frac{15}{4}=\frac{1\cdot2\cdot5}{1\cdot2\cdot4}=\frac{5}{4}$

$$(d) \quad \frac{1}{3} \left(\frac{9}{20} \right) + \frac{5}{6} = \frac{3}{20} + \frac{5}{6} = \frac{3}{20} \left(\frac{3}{3} \right) + \frac{5}{6} \left(\frac{10}{10} \right) = \frac{9+50}{60} = \frac{59}{60}$$

8. Find the GCF and LCM of the following pairs of numbers. (You can leave your answers in factored form, if you'd like.)

(a) 210, 300 Answer: GCF = 30 LCM = 2100

(b) 110, 198 Answer: GCF = 22, LCM = 990

9. Perform the following arithmetic operations. For each problem, use one method to show your work. (For example, you can use the chip abacus, base blocks, number line, etc.) (Note: Make sure you can do both partitive and measurement methods for division.)

(a) $6 - 4 = 2$

(b) $\frac{3}{4} + \frac{1}{5} = \frac{3}{4} \left(\frac{5}{5} \right) + \frac{1}{5} \left(\frac{4}{4} \right) = \frac{15+4}{20} = \frac{19}{20}$

(c) $\frac{5}{8} \div \frac{1}{2} = \frac{5}{8} \cdot \frac{2}{1} = \frac{5}{4}$

(d) $561 \div 8 = 70 r 1$

(e) $5209 \times 67 = 349,003$

(f) $-3 + 10 = 7$

(g) $\frac{2}{3} \cdot \frac{1}{6} = \frac{1 \cdot 1}{3 \cdot 3} = \frac{1}{9}$

(h) $-5 \times -4 = 20$

(i) $238 + 96 = 334$

(j) $177 - 85 = 92$

10. Solve these linear equations using (i) a diagram and (ii) algebra notation.

(a) $-2x + 9 = 4x - 6 + (-x)$
 $-2x + 9 = 3x - 6$
 $15 = 5x$
 $3 = x$

$$\begin{aligned}
 \text{(b)} \quad & 3(x-2) + 5 = -2x + 4 \\
 & 3x - 6 + 5 = -2x + 4 \\
 & 3x - 1 = -2x + 4 \\
 & 5x = 5 \\
 & x = 1
 \end{aligned}$$

11. Solve the systems of equations.

$$\text{(a)} \quad \begin{array}{l} 2x + 3y = 1 \\ 5x - y = 11 \end{array} \quad \text{Answer: } (2, -1)$$

$$\text{(b)} \quad \begin{array}{l} -3x + y = 7 \\ 2y = 6x + 14 \end{array} \quad \text{Answer: same lines (infinitely many points in common)}$$

$$\text{(c)} \quad \begin{array}{l} x - 4y = 18 \\ 3x + 2y = 12 \end{array} \quad \text{Answer: } (6, -3)$$

$$\text{(d)} \quad \begin{array}{l} 2x + y = 7 \\ -5x - 3y = -21 \end{array} \quad \text{Answer: } (0, 7)$$

$$\text{(e)} \quad \begin{array}{l} 2x + 8y = 9 \\ 2 + x = -4y \end{array} \quad \text{Answer: No solution (the lines are parallel)}$$

12. Solve the quadratic equations.

$$\begin{aligned}
 & x^2 - 16 = 0 \\
 \text{(a)} \quad & x^2 = 16 \\
 & x = \pm \sqrt{16} \\
 & x = \pm 4
 \end{aligned}$$

$$\begin{aligned}
 & x^2 + 2x - 15 = 0 \\
 \text{(b)} \quad & (x+5)(x-3) = 0 \\
 & x+5=0 \text{ or } x-3=0 \\
 & x = -5, 3
 \end{aligned}$$

$$\begin{aligned}
 & 2(x+3)^2 - 50 = 0 \\
 & 2(x+3)^2 = 50 \\
 \text{(c)} \quad & (x+3)^2 = 25 \\
 & x+3 = \pm 5 \\
 & x+3 = -5 \text{ or } x+3 = 5 \\
 & x = -8, 2
 \end{aligned}$$

$$\begin{aligned}
 2x^2 - 35 &= -9x \\
 2x^2 + 9x - 35 &= 0 \\
 \text{(d)} \quad (2x - 5)(x + 7) &= 0 \\
 x &= \frac{5}{2}, -7
 \end{aligned}$$

13.If possible, find an odd number that can be expressed as the sum of four consecutive counting numbers. If impossible, explain why.

Answer: The sum of four consecutive counting numbers is $n + (n + 1) + (n + 2) + (n + 3)$ which simplifies to $4n + 6 = 2(2n + 3)$ which is always even. So, there is no way we can get an odd number.

14.Think of a number. Multiply it by 5. Add 8. Multiply by 4. Add 9. Multiply by 5. Subtract 105. Divide by 100. Subtract 1. How does your result compare with your original number? Explain why.

$$\begin{aligned}
 \text{Answer: } & \frac{5(4(5x+8)+9)-105}{100} - 1 = \frac{5(20x+32+9)-105}{100} - 1 = \frac{5(20x+41)-105}{100} - 1 \\
 = & \frac{100x+205-105}{100} - 1 = \frac{100x+100}{100} - 1 = \frac{100(x+1)}{100} - 1 = x+1-1 = x
 \end{aligned}$$

Your answer is always the original number you started with.

15.Together a baseball and a football weigh 1.25 pounds, the baseball and a soccer ball weigh 1.35 pounds, and the football and the soccer ball weigh 1.6 pounds. How much does each of the balls weigh? Explain your reasoning.

Answer: soccer ball 0.85 pound, football 0.75 pound, baseball 0.5 pound

16.In three years, mike will be three times my present age. I will then be half as old as he. How old am I now?

Answer: I am 6 years old now.

17.When Betty sorts her marbles, she notices that if she puts them into groups of 5, she has 1 left over. When she puts them in groups of 7, she also has 1 left over, but in groups of size 6, she has none left over. What is the smallest number of marbles she could have?

Answer: 36

18.An auditorium contains 315 occupied seats and was $\frac{7}{9}$ filled. How many empty seats were there?

Answer: 405 seats total, $405 - 315 = 90$ seats empty

19. If $3\frac{1}{4}$ cups of sugar are used to make a batch of candy for 30 people, how many cups are required for 55 people?

Answer: $5\frac{23}{24}$ cups

20. The bill (including parts and labor) for the repair of a home appliance was \$165. The cost for parts was \$85. The labor rate is \$16 per half hour. How many hours were spent in repairing the appliance?

Answer: 2.5 hours

21. A store is offering a 20% discount on all items in its inventory. Find the list price on a tractor that has a sale price of \$6400.

Answer: \$8000

22. A pet store owner mixes two types of dog food costing \$1.50 per pound and \$3.05 per pound to make 25 pounds of a mixture costing \$2.12 per pound. How many pounds of each kind of dog food are in the mixture?

Answer: 15 pounds of \$1.50/lb variety and 10 pounds of the other

23. A fundraising dinner was held on two consecutive nights. ON the first night, 100 adult tickets and 175 children's tickets were sold, for a total of \$937.50. On the second night, 200 adult tickets and 316 children's tickets were sold, for a total of \$1790.00. Find the price of each type of ticket.

Answer: Adult ticket price \$5, child ticket price \$2.50

24. The perimeter of a rectangle is 68 feet and its width is $\frac{8}{9}$ times its length.

Find the dimensions of the rectangle.

Answer: length 18 feet, width 16 feet

25. If $\frac{3}{5}$ of a pound of nuts costs \$3.60, how much does $2\frac{1}{3}$ pounds of nuts cost?

Answer: \$14.00

26. In parenting a teenage daughter, I've discovered that patience is a necessary commodity. During the school year, $\frac{1}{3}$ of my patience was used up in the first term. The second term required $\frac{2}{5}$ of the patience that I had left. In the third term, I used up $\frac{3}{4}$ of my remaining patience. How much patience did I have left for the fourth term?

Answer: 1/10 of my original amount of patience