Key Definitions: Section 2.1

• The identity matrix $I_n$ is

• A diagonal matrix is

• A zero matrix is

• The transpose of a matrix $A$ is
Theorem 1 Properties of Matrix Addition and Scalar Multiplication Let $A$, $B$, and $C$ be matrices of the same size, $m \times n$, and let $r$ and $s$ be scalars.

(a) $A + B =$
(b) $(A + B) + C =$
(c) $A + 0 =$
(d) $r(A + B) =$
(e) $(r + s)A =$
(f) $r(sA) =$

Theorem 2 Properties of Matrix Multiplication Let $A, B,$ and $C$ be matrices and $r$ be a scalar such that the sums and products below are defined. Then,

(a) $A(BC) =$
(b) $A(B + C) =$
(c) $(B + C)A =$
(d) $r(AB) =$
(e) $I_mA =$

Theorem 3 Transpose Properties Let $A$ and $B$ be matrices whose sizes are appropriate for the following sums and products.

(a) $(A^T)^T =$
(b) $(A + B)^T =$
(c) For any scalar $r$, $(rA)^T =$
(d) $(AB)^T =$

Note: The transpose of a product of matrices equals the product of their transposes in reverse order.
Supplemental Practice Problems:

1. Consider the following matrices

\[
A = \begin{bmatrix}
2 & -1 & 4 \\
-3 & 1 & 2
\end{bmatrix}, \quad B = \begin{bmatrix}
1 & -2 \\
2 & 5 \\
3 & 3
\end{bmatrix}, \quad \begin{bmatrix}
2 & -5 \\
-1 & 3
\end{bmatrix}
\]

Calculate (if possible) each of the following matrix products:

(a) \(AB\)  \hspace{1cm} (c) \(AC\)  
(b) \(BA\)  \hspace{1cm} (d) \(CA\)

2. Let \(R\) be the rectangle with vertices \((-2, -1), (-2, 2), (3, 2), (3, -1)\). Consider the linear transformation \(T: \mathbb{R}^2 \to \mathbb{R}^2\) given by \(T(x_1, x_2) = (4x_1 - 3x_2, -x_1 + x_2)\).

Find the standard matrix \(A\) of the linear transformation \(T\), and sketch the image of the rectangle \(R\) under \(T\).

3. Let \(R\) be the rectangle with vertices \((-2, -1), (-2, 2), (3, 2), (3, -1)\). Consider the linear transformation \(S: \mathbb{R}^2 \to \mathbb{R}^2\) which maps the unit square to the parallelogram pictured below.

Find the standard matrix \(B\) associated to \(S\).

4. Consider the matrices \(A = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}\) and \(B = \begin{bmatrix} 1 & b \\ c & -1 \end{bmatrix}\) where \(b\) and \(c\) are unknowns. Find values of \(b\) and \(c\) such that \(AB = BA\).